CALCULUS

DIFFERENTIATION

WORKED EXAMPLES

QUESTIONS AND SOLUTIONS

PRODUCT AND QUOTIENT RULES

Note.

(1). The Product Rule

If *u = f(x)* and *v = g(x)* are differentiable, then $(fg)' = f'g + fg'$

(2). The Quotient Rule.

If *u = f(x)* and *v = g(x)* are differentiable, then $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Find the derivative of the following using the product or quotient rule.

1) $f(x) = x^2(x^3+5)$

2) $f(x) = 2^x * 3^x$

3) $f(x) = xe^x$

4) $y = x*2^x$

5) $y = \sqrt{x}*2^x$

6) $f(x) = (x^2 - \sqrt{x})3^x$

7) $z = (s^2 - \sqrt{s})(s^2 + \sqrt{s})$

8) $y = (t^2+3)e^3$

9) $g = (t^3 - 7t^2 + 1)e^t$

10) $f(x) = \frac{x}{e^x}$

11) $g(x) = \frac{25x^2}{e^x}$

12) $g(w) = \frac{w^{3.2}}{5^w}$

13) $q(r) = \frac{3r}{5r+2}$

14) $g(t) = \frac{t-4}{t+4}$

15) $z = \frac{3t+1}{5t+2}$

16) $z = \frac{t^2+5t+2}{t+3}$

17) $z = \frac{t^2+3t+1}{t+1}$

18) $f(x) = \frac{x^2+3}{3}$

19) $w = \frac{y^3 - 6y^2 + 7y}{y}$

20) $y = \frac{\sqrt{t}}{t^2+1}$

21) $f(z) = \frac{z^2+1}{\sqrt{z}}$

22) $g(t) = \frac{4}{3+\sqrt{t}}$

23) $h(r) = \frac{r^2}{2r+1}$

24) $f(x) = \frac{3z^2}{5z^2+7z}$

25) $w(x) = \frac{17e^x}{2^x}$

26) $h(p) = \frac{1+p^2}{3+2p^2}$

27) $f(x) = \frac{1+x}{2+3x+4x^2}$

28) $f(x) = \frac{ax+b}{cx+k}$

29) $w = (t^3+5t)(t^2-7t+2)$

30) $f(x) = (3x+8)(2x-5)$

SOLUTIONS

1) $f(x)=x^2(x^3+5)$

$f'(x)=2x*(x^3+5)+x^2(3x^2)$

$=2x^4+10x+3x^4 \Rightarrow 5x^4+10x$

2) $f(x)=2^x*3^x$

$f'(x)=2^x(\ln 3)3^x+2^x(\ln 2)3^x$

$= 2^x*3^x(\ln 2+\ln 3)$

$= 6^x(\ln 6)$

3) $f(x)=xe^x \Rightarrow xe^x+e^x$

$f'(x)=e^x(x+1)$

4) $y=x*2^x$

$y'=x(\ln 2)2^x+2^x$

$=2^x(1+x\ln 2)$

5) $y=\sqrt{x}*2^x \Rightarrow x^{\frac{1}{2}}*2^x$

$y'=x^{\frac{1}{2}}(\ln 2)2^x+\frac{1}{2}x^{-\frac{1}{2}}2^x$

$=x^{\frac{1}{2}}(\ln 2)2^x+\frac{2^x}{2x^{\frac{1}{2}}}$

6) $f(x)=(x^2-\sqrt{x})3^x \Rightarrow x^2-x^{\frac{1}{2}}$

$f'(x)=\left(x^2-x^{\frac{1}{2}}\right)3^x\ln 3+\left(2x-\frac{1}{2}x^{-\frac{1}{2}}\right)3^x$

$=3^x\ln 3\left(x^2-x^{\frac{1}{2}}\right)+\left(2x-\frac{1}{2x^{\frac{1}{2}}}\right)$

7) $z=(s^2-\sqrt{s})(s^2+\sqrt{s})$

$\Rightarrow\left((s^2)^2-\left(s^{\frac{1}{2}}\right)^2\right)\Rightarrow s^4-s$

$z'=4s^3-1$

8) $y=(t^2+3)e^t$

$y'=2te^t+e^t(t^2+3)$

$=e^t(t^2+2t+3)$

9) $g=(t^3-7t^2+1)e^t$

$g'=(3t^2-14t)e^t+(t^3-7t^2+1)e^t$

$=e^t(t^3-4t^2-14t+1)$

10) $f(x)=\frac{x}{e^x}$

$f'(x)=\frac{e^x-xe^x}{(e^x)^2}\Rightarrow\frac{e^x(1-x)}{(e^x)^2}\Rightarrow\frac{1-x}{e^x}$

11) $g(x)=\frac{25x^2}{e^x}$

$g'(x)=\frac{50xe^x-25x^2e^x}{(e^x)^2}$

$=\frac{e^x(50x-25x^2)}{(e^2)^2}$

$=\frac{50x-25x^2}{e^x}$

12) $g(w)=\frac{w^{3.2}}{5^w}$

$g'(w)=\frac{3.2w^{2.2}(5^w)-w^{3.2}(5^w)\ln 5}{(5^w)^2}$

$=\frac{5^w(3.2w^{2.2}-w^{3.2}\ln 5)}{(5^w)^2}$

$=\frac{3.2w^{2.2}-w^{3.2}\ln 5}{5^w}$

13) $q(r)=\dfrac{3r}{5r+2}$

$$q'(r)=\frac{3(5r+2)-3r*5}{(5r+2)^2}$$

$$=\frac{15r+6-15r}{(5r+2)^2}\Rightarrow\frac{6}{(5r+2)^2}$$

14) $g(t)=\dfrac{t-4}{t+4}$

$$g'(t)=\frac{(1)(t+4)-(t-4)(1)}{(t+4)^2}$$

$$=\frac{t+4-t+4}{(t+4)^2}\Rightarrow\frac{8}{(t+4)^2}$$

15) $z=\dfrac{3t+1}{5t+2}$

$$z'=\frac{3(5t+2)-5(3t+1)}{(5t+2)^2}$$

$$=\frac{15t+6-15t-5}{(5t+2)^2}$$

$$=\frac{1}{(5t+2)^2}$$

16) $z=\dfrac{t^2+5t+2}{t+3}$

$$z'=\frac{(2t+5)(t+3)-(1)(t^2+5t+2)}{(t+3)^2}$$

$$=\frac{2t^2+6t+5t+15-t^2-5t-2}{(t+3)^2}$$

$$=\frac{t^2+6t+13}{(t+3)^2}$$

17) $z=\dfrac{t^2+3t+1}{t+1}$

$$z'=\frac{(2t+3)(t+1)-(1)(t^2+3t+1)}{(t+1)^2}$$

$$=\frac{2t^3+2t+3t+3-t^2-3t-1}{(t+1)^2}$$

$$=\frac{t^2+2t+2}{(t+1)^2}$$

18) $f(x)=\dfrac{x^2+3}{x}$

$$f'(x)=\frac{(2x)x-(1)(x^2+3)}{x^2}$$

$$=\frac{2x^2-x^2-3}{x^2}\Rightarrow\frac{x^2-3}{x^2}$$

19) $w=\dfrac{y^3-6y^2+7y}{y}\Rightarrow y^2-6y+7$

$w'=2y-6,\ y\neq 0$

20) $y=\dfrac{\sqrt{t}}{t^2+1}\Rightarrow\dfrac{t^{\frac{1}{2}}}{t^2+1}$

$$y'=\frac{\frac{1}{2}t^{-\frac{1}{2}}(t^2+1)-(2t)t^{\frac{1}{2}}}{(t^2+1)^2}$$

$$=\frac{\frac{1}{2\sqrt{t}}(t^2+1)-\sqrt{t}(2t)}{(t^2+1)^2}$$

21) $f(z)=\dfrac{z^2+1}{\sqrt{z}}$

$$f'(z)=\frac{2z\left(z^{\frac{1}{2}}\right)-(z^2+1)\left(\frac{1}{2}z^{-\frac{1}{2}}\right)}{z}$$

$$=\frac{2z^{\frac{3}{2}}-\left(\frac{1}{2}z^{\frac{3}{2}}+\frac{1}{2}z^{-\frac{1}{2}}\right)}{z}$$

$$=\frac{\frac{3}{2}z^{\frac{3}{2}}-\frac{1}{2}z^{-\frac{1}{2}}}{z}\Rightarrow\frac{3z^{\frac{3}{2}}-z^{-\frac{1}{2}}}{2z}$$

22) $g(t)=\dfrac{4}{3+\sqrt{t}} \Rightarrow 4\left(3+t^{\frac{1}{2}}\right)^{-1}$

$g'(t)=-4\left(3+t^{\frac{1}{2}}\right)^{-2}\left(\dfrac{1}{2}t^{-\frac{1}{2}}\right)$

$=\dfrac{-4*\dfrac{1}{2}}{\left(3+\sqrt{t}\right)^{2}\left(\sqrt{t}\right)} \Rightarrow \dfrac{-2}{\left(3+\sqrt{t}\right)^{2}\left(\sqrt{t}\right)}$

23) $h(r)=\dfrac{r^2}{2r+1}$

$h'(r)=\dfrac{(2r+1)(2r)-2r^2}{(2r+1)^2}$

$=\dfrac{4r^2+2r-2r^2}{(2r+1)^2}$

$=\dfrac{2r^2+2r}{(2r+1)^2} \Rightarrow \dfrac{2r(r+1)}{(2r+1)^2}$

24) $f(x)=\dfrac{3z^2}{5z^2+7z} \Rightarrow \dfrac{3z}{5z-7}$

$f'(x)=\dfrac{3(5z+7)-5(3z)}{(5z+7)^2}$

$=\dfrac{15z+21-15z}{(5z+7)^2} \Rightarrow \dfrac{21}{(5z+7)^2}$

25) $w(x)=\dfrac{17e^x}{2^x}$

$w'(x)=\dfrac{\left(2^x\right)\left(17e^x\right)-\left(2^x\right)\ln 2\left(17e^x\right)}{\left(2^x\right)^2}$

$=\dfrac{17e^x(2^x)\left(1-(\ln 2)\right)}{\left(2^x\right)^2}$

$=\dfrac{17e^x\left(1-\ln 2\right)}{2^x}$

26) $h(p)=\dfrac{1+p^2}{3+2p^2}$

$h'(p)=\dfrac{2p\left(3p+2p^2\right)-4p\left(1+p^2\right)}{\left(3+2p^2\right)^2}$

$=\dfrac{6p+4p^3-4p-4p^3}{(3+2p)}$

$=\dfrac{2p}{\left(3+2p^2\right)^2}$

27) $f(x)=\dfrac{1+x}{2+3x+4x^2}$

$f'(x)=\dfrac{(1)\left(2+3x+4x^2\right)-(1+x)(8x+3)}{(2+3x+4x^2)^2}$

$=\dfrac{\left(2+3x+4x^2\right)-\left(8x+3+8x^2+3x\right)}{\left(2+3x+4x^2\right)^2}$

$=\dfrac{-1-8x-4x^2}{\left(2+3x+4x^2\right)^2}$

28) $f(x)=\dfrac{ax+b}{cx+k}$

$f'(x)=\dfrac{a(cx+k)-c(ax+b)}{(cx+k)^2}$

$=\dfrac{(acx+ak)-(acx+bc)}{(cx+k)^2}$

$=\dfrac{ak-bc}{(cx+k)^2}$

29) $w=\left(t^3+5t\right)\left(t^2-7t+2\right)$

$w'=\left(3t^2+5\right)\left(t^2-7t+2\right)+\left(t^3+5t\right)(2t-7)$

30) $f(x)=(3x+8)(2x-5)$

$f'(x)=3(2x-5)+2(3x+8)$

$=6x-15+6x+16 \Rightarrow 12x+1$