CONTINUITY

In **calculus**, a function is continuous at x = a, **iff** all three of the following conditions are met:

(a) The function is defined at x = a; that is, f(a) equals a real number.

(b) The limit of the function as x approaches a exists.

(c) The limit of the function as x approaches a is equal to the function value at x = a.

Put simply, a continuous function is one that could be drawn without lifting your pencil from the paper, i.e. one with no gaps. Put another way, a function is continuous if there are no breaks, no holes, or asymptotes.

Determine if each function is continuous at the given *x*-value. If not continuous, classify each discontinuity.

1. $y=\frac{x+1}{|x+1|}$; at x =1 and -1
2. $f(x)=\frac{x+2}{x^2-4}$; at x = 2 and x = -2
3. $f(x)=\frac{x^2}{x+1}$; at x = -1
4. $f(x)=\frac{x^2+4x+3}{x+3}$; at x = 3 and x = -3
5. $f(x)=\frac{x^2-x-2}{x+1}$; at x =1and x = -1
6. $f(x)=\frac{x-3}{x^2-x}$; at x = 0 and x = 3

Find the interval on which each function is continuous.

7. $f(x)=\frac{x-1}{x^2-x}$
8. $f(x)=\begin{cases}x^2-2x+2, x<1\\2x-1, x\geq 1\end{cases}$
9. $f(x)=\begin{cases}x^2+2x+1, x<1\\-\frac{1}{2}x, \geq 1\end{cases}$
10. $f(x)=\begin{cases}x^2-4x+3, x\neq 0\\3, x=0\end{cases}$
11. $f(x)=\begin{cases}2x-10, x<2\\0, x\geq 2\end{cases}$
12. $f(x)=\frac{x^2-x-12}{x+3}$
13. $f(x)=\frac{x^2-x-6}{x+2}$
14. $f(x)=\frac{x+1}{x^2+x+1}$
15. $f(x)=\begin{cases}x, x<-1\\-x^2+2x, x\geq -1\end{cases}$

Solutions.

1. $y = \frac{x+1}{|x+1|}$; at x =1 and -1. Discontinuous at x = -1.

2. $f(x) = \frac{x+2}{x^2-4}$; at x = 2 and x = -2. Removable discontinue at x = -2, Infinite discontinuity at x = 2.

3. $f(x) = \frac{x^2}{x+1}$; at x = -1. Infinite Discontinuity at x = -1

4. $f(x) = \frac{x^2+4x+3}{x+3}$; at x =3 and x = - 3. Removable Discontinuity at x = -3

5. $f(x) = \frac{x^2-x-2}{x+1}$; at x =1and x = -1. Removable Discontinuity at x = -1

6. $f(x) = \frac{x-3}{x^2-x}$; at x = 0 and x = 3. Infinite Discontinuity at x = 0, 1

Find the interval on which each function is continuous.

7. $f(x) = \frac{x-1}{x^2-x}$. Answer: $(-\infty, 0), (0,1), (1, \infty)$

8. $f(x) = \begin{cases} x^2-2x+2, x<1 \\ 2x-1, x \geq 1 \end{cases}$ Answer: $(-\infty, 0), (1, \infty)$

9. $f(x) = \begin{cases} x^2+2x+1, x<1 \\ -\frac{1}{2}x, \geq 1 \end{cases}$ Answer: $(-\infty, \infty), [1, \infty)$

10. $f(x) = \begin{cases} x^2-4x+3, x \neq 0 \\ 3, x=0 \end{cases}$ Answer: $(-\infty, \infty)$

11. $f(x) = \begin{cases} 2x-10, x<2 \\ 0, x \geq 2 \end{cases}$ Answer: $(-\infty, \infty)$

12. $f(x) = \frac{x^2-x-12}{x+3}$ Answer: $(-\infty, \infty)$

13. $f(x) = \frac{x^2-x-6}{x+2}$ Answer: $(-\infty, \infty)$

14. $f(x) = \frac{x+1}{x^2+x+1}$ Answer $(-\infty, \infty)$

15. $f(x) = \begin{cases} x, x<-1 \\ -x^2+2x, x \geq -1 \end{cases}$ Answer: $(-\infty, -1), [-1, \infty)$