CALCULUS 1

UNIT 3

THE DERIVATIVE

LESSON 6

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IMPLICIT DIFFERENTIATION

Implicit differentiation is the procedure of **differentiating** an **implicit** equation with respect to the desired variable x while treating the other variables as unspecified functions of x. It is a technique based on the Chain Rule that is used to find a derivative when the relationship between the variables is given implicitly rather than explicitly (solved for one variable in terms of the other). There are two ways to define functions, *implicitly* and *explicitly*. Most of the equations we have dealt with have been explicit equations, such as $y = x^2 + 2$. But the equation $x^2 - y = -2$. describes the same function.

Use implicit differentiation to find *dy/dx* in terms of *x* and *y*.

Example 1:

$10x^4 - 18xy^2 + 10y^3 = 48$

Solution.

$$40x^3 - 18y^2 - 36xyy' + 30y^2y' = 0$$

$$-36xyy' + 30y^2y' = 18y^2 - 40x^3$$

$$y'(-36xy + 30y^2) = 18y^2 - 40x^3$$

$$y' = \frac{18x^3 - 40x^3}{30y^2 - 36xy} = \frac{2(9y^2 - 20x^3)}{6(5y^2 - 6xy)} = \frac{9y^2 - 20x^3}{3(5y^2 - 6xy)}$$

Example 2:

$x^3 + x^2y + 4y^2 = 6$

Solution.

$$3x^2 + 2xy + x^2y' + 8yy' = 0$$

$$x^2y' + 8yy' = -3x^2 - 2xy$$

$$y'(x^2 + 8y) = -3x^2 - 2xy$$

$$y' = \frac{-3x^2 - 2xy}{x^2 + 8y} = \frac{-x(3x + 2y)}{x^2 + 8y}$$

Example 3:

$x^2y + xy^2 = 3x \Rightarrow x^2y + xy^2 - 3x = 0$

Solution.

$$2xy + x^2y' + y^2 + 2xyy' - 3 = 0$$

$$2xy + y^2 - 3 + x^2y' + 2xyy' = 0$$

$$x^2y' + 2xyy' = 3 - 2xy - y^2$$

$$y'(x^2 + 2xy) = 3 - 2xy - y^2$$

$$y' = \frac{3 - 2xy - y^y}{x^2 + 2xy}$$

Example 4:

$x^3 + y^3 = 6xy \Rightarrow x^3 + y^3 - 6xy = 0$

Solution.

$$3x^2+3yy'-6y-6xy'=0$$

$$3x^2-6y+3y^2y'-6xy'=0$$

$$3y^2y'-6xy'=6y-3x^2$$

$$y'\left(3y^2-6x\right)=6y-3x^2$$

$$y'=\frac{6y-3x^2}{3y^2-6x}=\frac{3\left(2y-x^2\right)}{3\left(y^2-2x\right)}$$

$$y'=\frac{2y-x^2}{y^2-2x}$$

Exercise

1) $x^2y+xy^2=6$
2) $x^2-xy=5$
3) $3x^2+3y^2=2$
4) $x^3y^3-y=x$
5) $x^2+xy-y^2=1$
6) $5x^3=-3xy+2$
7) $3x^2y^2=4x^2-4xy$
8) $5x^3+xy^2=5x^3y^3$
9) $2y^3+4x^2-y=x^6$

SOLUTIONS

1) $x^2y+xy^2=6$

$$2xy+x^2y'+y^2+2xyy'=0$$

$$2xy+y^2+x^2y'+2xyy'=0$$

$$x^2y'+2xyy'=-2xy-y^2$$

$$y'\left(x^2+2xy\right)=-2xy-y^2$$

$$y'=-\frac{2xy+y^2}{x^2+2xy}$$

2) $x^2-xy=5$

$$2x-y+xy'=0$$

$$xy'=2x-y$$

$$y'=\frac{2x-y}{x}$$

3) $3x^2+3y^2=2$

$$6x+6yy'=0$$

$$6yy'=-6x$$

$$y'=-\frac{6x}{6y}=-\frac{x}{y}$$

4) $x^3y^3 - y = x \Rightarrow x^3y^3 - y - x = 0$

$3x^2y^3 + 3x^3y^2y' - y' - 1 = 0$

$3x^3y^2y' - y' = -3x^2y^3 + 1$

$y'\left(3x^3y^2 - 1\right) = 1 - 3x^2y^3$

$y' = \dfrac{1 - 3x^2y^3}{3x^3y^2 - 1} =$

5) $x^2 + xy - y^2 = 1$

$2x + y + xy' - 2yy' = 0$

$xy' - 2yy' = -2x - y$

$y'(x - 2y) = -2x - y$

$y' = \dfrac{-2x - y}{x - 2y}$

6) $5x^3 = -3xy + 2 \Rightarrow 5x^3 + 3xy = 2$

$15x^2 + 3y + 3xy' = 0$

$3xy' = -15x^2 - 3y$

$y' = \dfrac{-15x^2 - 3y}{3x} = \dfrac{-5x^2 - y}{x}$

7) $3x^2y^2 = 4x^2 - 4xy \Rightarrow 3x^2y^2 - 4x^2 + 4xy = 0$

$6xy^2 + 6x^2yy' - 8x + \left(4y + 4xy'\right) = 0$

$6xy^2 + 6x^2yy' - 8x + 4y + 4xy' = 0$

$6x^2yy' + 4xy' = -6xy^2 + 8x - 4y$

$y'\left(6x^2y + 4x^2\right) = 8x - 4y - 6xy^2$

$y' = \dfrac{8x - 4y - 6xy^2}{6x^2y + 4x} = \dfrac{4x - 2y - 3y}{3x^2y + 2x}$

8) $5x^3 + xy^2 = 5x^3y^3 \Rightarrow 5x^3 + xy^2 - 5x^3y^3 = 0$

$15x^2 + y^2 + 2xyy' - \left(15x^2y^3 + 15x^3y^2y'\right) = 0$

$15x^2 + y^2 + 2xyy' - 15x^2y^3 - 15x^3y^2y' = 0$

$2xyy' - 15x^3y^2y' = -15x^2 - y^2 + 15x^2y^3$

$y'\left(2xy - 15x^3y^2\right) = 15x^2y^3 - 15x^2 - y^2$

$y' = \dfrac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$

9) $2y^3 + 4x^2 - y = x^6 \Rightarrow 2y^3 + 4x^2 - y - x^6 = 0$

$6y^2y' + 8x - y' - 6x^5 = 0$

$6y^2y' - y' = 6x^5 - 8x$

$y'\left(6y^2 - 1\right) = 6x^5 - 8x$

$y' = \dfrac{6x^5 - 8x}{6y^2 - 1}$