CALCULUS 1

UNIT 3

THE DERIVATIVE

LESSON 4

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THE QUOTIENT RULE

The Quotient Rule.

The Quotient Rule says that the derivative of a quotient is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Then,

If *u = f(x)* and *v = g(x)* are differentiable, then $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

The quotient rule in calculus is a method used to find the derivative of any function given in the form of a quotient obtained from the result of the division of two differentiable functions. The quotient rule in words states that the derivative of a quotient is equal to the ratio of the result obtained on the subtraction of the numerator times the derivative of the denominator from the denominator times the derivative of the numerator to the square of the denominator. That means if we are given a function of the form: $f(x) = \frac{u(x)}{v(x)}$ we can find the derivative of this function using the quotient rule derivative given above.

The following examples demonstrate how the rule is applied.

Find the derivative of the following using the quotient rule.

Example 1

$$f(x) = \frac{x^2 + 6}{2x - 7}$$

$$f'(x) = \frac{2x(2x - 7) - 2\left(x^2 + 6\right)}{(2x - 7)^2}$$

$$= \frac{4x^2 - 14x - 2x^2 - 12}{(2x - 7)^2} = \frac{2x^2 - 14x - 12}{(2x - 7)^2}$$

$$= \frac{2\left(x^2 - 7x - 6\right)}{(2x - 7)^2}$$

Example 2

$$y = \frac{4x^3 - 3x^2}{4x^5 - 4}$$

$$y' = \frac{\left(12x^2 - 6x\right)\left(4x^5 - 4\right) - 20x^4\left(4x^3 - 3x^2\right)}{\left(4x^5 - 4\right)^2}$$

$$= \frac{48x^7 - 48x^2 - 24x^6 + 24x - 80x^7 + 60x^6}{\left(4x^5 - 4\right)^2}$$

$$= \frac{-32x^7 + 36x^6 - 48x^2 + 24x}{16x^{10} - 32x^5 + 16}$$

$$= \frac{-8x^7 + 9x^6 - 12x^2 + 6x}{4x^{10} - 8x^5 + 4}$$

Example 3

$$y = \frac{4x^3 + 2x}{x - 6}$$

$$y' = \frac{\left(12x^2 + 2\right)(x - 6) - (1)\left(4x^3 + 2x\right)}{(x - 6)^2}$$

$$= \frac{12x^3 - 72x^2 + 2x - 12 - 4x^3 - 2x}{(x - 6)^2}$$

$$= \frac{8x^3 - 72x^2 - 12}{(x - 6)^2}$$

Example 4

$$y = \frac{x^3 - x^2 - 3}{x^5 + 3}$$

$$y' = \frac{\left(3x^2 - 2x\right)\left(x^5 + 3\right) - 5x^4\left(x^3 - x^2 - 3\right)}{\left(x^5 + 3\right)^2}$$

$$= \frac{3x^7 + 9x^2 - 2x^6 - 6x - 5x^7 + 5x^6 + 15x^4}{\left(x^5 + 3\right)^2}$$

$$= \frac{-2x^7 + 3x^6 + 15x^4 + 9x^2 - 6x}{\left(x^5 + 3\right)^2}$$

Lesson 4 Exercise

Find the derivative of the following using the quotient rule.

1. $q(r) = \frac{3r}{5r + 2}$

2. $g(t) = \frac{t - 4}{t + 4}$

3. $z = \frac{3t + 1}{5t + 2}$

4. $z = \frac{t^2 + 5t + 2}{t + 3}$

5. $z = \frac{t^2 + 3t + 1}{t + 1}$

6. $f(x) = \frac{x^2 + 3}{x}$

7. $w \square \frac{y^3 - 6y^2 \square 7y}{y}$

8. $h(r) = \frac{r^2}{2r + 1}$

9. $f(x) = \frac{3z^2}{5z^2 + 7z}$

10. $h(p) = \frac{1 + p^2}{3 + 2p^2}$

11. $f(x) = \frac{1 + x}{2 + 3x + 4x^2}$

$$y=\frac{3x^4+5x^3-5}{2x^4-4}$$

SOLUTIONS

1) $q(r)=\frac{3r}{5r+2}$

$$q'(r)=\frac{3(5r+2)-5(3r)}{(5r+2)^2}$$

$$=\frac{15r+6-15r}{(5r+2)^2}=\frac{6}{(5r+2)^2}$$

2) $g(t)=\frac{t-4}{t+4}$

$$g'(t)=\frac{t-4}{(t+4)^2}=\frac{1(t+4)-1(t-4)}{(t+4)^2}$$

$$=\frac{t+4-t+4}{(t+4)^2}=\frac{8}{(t+4)^2}$$

3) $z=\frac{3t+1}{5t+2}$

$$z'=\frac{3(5t+2)-5(3t+1)}{(5t+2)^2}$$

$$=\frac{15t+6-15t-5}{(5t+2)^2}=\frac{1}{(5t+2)^2}$$

4) $z=\frac{t^2+5t+2}{t+3}$

$$z'=\frac{(2t+5)(t+3)-1\left(t^2+5t+2\right)}{(t+3)^2}$$

$$=\frac{2t^2+6t+5t+15-t^2-5t-2}{(t+3)^2}$$

$$=\frac{t^2+6t+13}{(t+3)^2}$$

5) $z=\frac{t^2+3t+1}{t+1}$

$$z'=\frac{(2t+3)(t+1)-(1)\left(t^2+3t+1\right)}{(t+1)^2}$$

$$=\frac{2t^2+2t+3t+3-t^2-3t-1}{(t+1)^2}$$

$$=\frac{t^2+2t+2}{(t+1)^2}$$

6) $f(x)=\frac{x^2+3}{x}$

$$f'(x)=\frac{x(2x)-1\left(x^2+3\right)}{x^2}$$

$$=\frac{2x^2-x^2-3}{x^2}=\frac{x^2-3}{x^2}$$

7) $w=\frac{y^3-6y^2+7y}{y}$

$$w'=\frac{\left(3y^2-12y+7\right)(y)-1\left(y^3-6y^2+7y\right)}{y^2}$$

$$=\frac{3y^3-12y^2+7y-y^3+6y^2-7y}{y^2}$$

$$=\frac{2y^3-6y^2}{y^2}=\frac{y^2(2y-6)}{y^2}=2y-6$$

8) $h(r)=\frac{r^2}{2r+1}$

$$h'(r)=\frac{2r(2r+1)-2\left(r^2\right)}{(2r+1)^2}$$

$$=\frac{4r^2+2r-2r^2}{(2r+1)^2}=\frac{2r^2-2r}{(2r+1)^2}$$

9) $f(x)=\frac{3z^2}{5z^2+7z}$

$$f'(z)=\frac{3(5z+7)-5(3z)}{(5z+7)^2}$$

$$=\frac{15z+21-15z}{(5z+7)^2}=\frac{21}{(5z+7)^2}$$

10) $h(p)=\frac{1+p^2}{3+2p^2}$

$$h'(p)=\frac{2p\left(3+2p^2\right)-4p\left(1+p^2\right)}{\left(3+2p^2\right)^2}$$

$$=\frac{6p+4p^3-4p-4p^3}{\left(3+2p^2\right)^2}=\frac{2p}{\left(3+2p^2\right)^2}$$

11) $f(x)=\dfrac{1+x}{2+3x+4x^2}$

$$f'(x)=\frac{(1)\left(2+3x+4x^2\right)-(1+x)(3+8x)}{\left(2+3x+4x^2\right)^2}$$

$$=\frac{2+3x+4x^2-3-8x-3x-8x^2}{\left(2+3x+4x^2\right)^2}$$

$$=\frac{-1-8x-4x^2}{\left(2+3x+4x^2\right)^2}$$

12) $y=\dfrac{3x^4+5x^3-5}{2x^4-4}$

$$y'=\frac{\left(12x^3+15x^2\right)\left(2x^4-4\right)-\left(3x^4+5x^3-5\right)\left(8x^3\right)}{\left(2x^4-4\right)^2}$$

$$=\frac{24x^7-48x^3+30x^6-60x^2-24x^7-40x^6+40x^3}{\left(2x^4-4\right)^2}$$

$$=\frac{-10x^6-8x^3-60x^2}{\left(2x^4-4\right)^2}=-\frac{2x^2\left(5x^4-4x+30\right)}{\left(2x^4-4\right)^2}$$