CALCULUS 1

UNIT 3

THE DERIVATIVE

LESSON 3

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THE PRODUCT RULE

The Product Rule

Note: $\{f(x) * g(x)\} = f(x) * g'(x) + f'(x) * g(x)$

What is the Product Rule?

The term "product" means that two functions are being multiplied together. This rule allows us to calculate derivatives that we don't want to multiply or are too many to multiply quickly. Put simply the rule says that the derivative of a product of two functions is:

The first times the derivative of the second plus the second times the derivative of the first.

To repeat, the **product rule** is a formal rule for differentiating problems where one function is multiplied by another function. To accomplish this, follow the formula given in the note above which is modeled in the examples shown below.

Find the derivative of the following using the product rule.

Example 1

$f(x) = (3x - 1)(x^2 + 2)$

$f'(x) = 3(x^2 + 2) + 2x(3x - 1)$

$= 3x^2 + 6 + 6x^2 - 2x$

$= 9x^2 - 2x + 6$

Example 2

$f(x) = 5x^2(2x^3 - 6)$

$f'(x) = 10x(2x^3 - 6) + 6x^2(5x^2)$

$= 20x^4 - 60x + 30x^4 = 50x^4 - 60x$

Example 3

$y = (8x^2 + 4)(x - 1)$

$y' = (16x)(x - 1) + (8x^2 + 4)(1)$

$= 16x^2 - 16x + 8x^2 + 4$

$= 24x^2 - 16x + 4$

Example 4

$h(z) = (2x^3 - 2)(4x + 3)$

$h'(z) = 6x^3(4x + 3) + 4(2x^3 - 2)$

$= 24x^3 + 18x^2 + 8x^3 - 8$

$= 32x^3 + 18x^2 - 8$

Lesson 3 Exercise

Find the derivative of the following using the product rule.

1) $f(x)=x^2(x^3+5)$
2) $f(t)=(4t^2-t)(t^3-8t+12)$
3) $f(x)=(x^3+x+1)(x^4+x+1)$
4) $w=(t^3+5t)(t^2-7t+2)$
5) $f(x)=(3x+8)(2x-5)$
6) $y=(x^2+1)(x^3+1)$
7) $y=-x^3(3x^4-2)$
8) $f(x)=x^2(-3x^2-2)$
9) $y=(-2x^4-3)(-2x^2+1)$
10) $f(x)=(2x^4-3)(x^2+1)$
11) $f(x)=(5x^5+5)(-2x^5-3)$
12) $y=(x^4+3)(-4x^5+5x^4+5)$

SOLUTIONS

1. $f(x)=x^2(x^3+5)$

$f'(x)=2x(x^3+5)+(x^2)(3x^2)$

$=2x^4+10x+3x^4$

$=5x^4+10x$

2. $f(t)=(4t^2-t)(t^3-8t+12)$

$f'(x)=(8t-1)(t^3-8t+12)+(4t^2-t)(3t^2-8)$

$=8t^4-64t^2+96t-t^3+8t-12+12t^4-32t^2-3t^3+8t$

$=20t^4-4t^3-96t^2+112t-12$

3. $f(x)=(x^3+x+1)(x^4+x+1)$

$f'(x)=(3x^2+1)(x^4+x+1)+(x^3+x+1)(4x^3+1)$

$=3x^6+3x^3+3x^2+x^4+x+1+4x^6+x^3+4x^4+x+4x^3+1$

$=7x^6+5x^4+8x^3+3x^2+2x+2$

4. $w=(t^3+5t)(t^2-7t+2)$

$w'=(3t^2+5)(t^2-7t+2)+(t^3+5t)(2t-7)$

$=3t^4-21t^3+6t^2+5t^2-35t+10+2t^4-7t^3+10t^2-35t$

$=5t^4-28t^3+21t^2-70t+10$

5. $f(x)=(3x+8)(2x-5)$

$f'(x)=3(2x-5)+(3x+8)(2)$

$=6x-15+6x+16$

$=12x+1$

6. $y=(x^2+1)(x^3+1)$

$y'=2x(x^3+1)+3x^2(x^2+1)$

$=2x^4+2x+3x^4+3x^2$

$=5x^4+3x^2+2x$

7. $y=-x^3(3x^4-2)$

$y'=-3x^2(3x^4-2)+(-x^3)(12x^3)$

$=-9x^6+6x^2-12x^6$

$=-21x^6+6x^2$

8. $f(x)=x^2(-3x^2-2)$

$f'(x) = 2x(-3x^2 - 2) + (x^2)(-6x)$

$= -6x^3 - 4x - 6x^3$

$= -12x^3 - 4x$

9. $y = (-2x^4 - 3)(-2x^2 + 1)$

$y' = -8x^3(-2x^2 + 1) + (-2x^4 - 3)(-4x)$

$= 16x^5 - 8x^3 + 8x^5 + 12x$

$= 24x^5 - 8x^3 + 12x$

10. $f(x) = (2x^4 - 3)(x^2 + 1)$

$f'(x) = 8x^3(x^2 + 1) + (2x^4 - 3)(2x)$

$= 8x^5 + 8x^3 + 4x^5 - 6x$

$= 12x^5 + 8x^3 - 6x$

11. $f(x) = (5x^5 + 5)(-2x^5 - 3)$

$f'(x) = 25x^4(-2x^5 - 3) + (5x^5 + 5)(-10x^4)$

$= -50x^9 - 75x^4 - 50x^9 - 50x^4$

$= -100x^9 - 125x^4$

12. $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

$y' = 4x^3(-4x^5 + 5x^4 + 5) + (x^4 + 3)(-20x^4 + 20x^3)$

$= -16x^8 + 20x^7 + 20x^3 - 20x^8 + 20x^7 - 60x^4 + 60x^3$

$= -36x^8 + 40x^7 - 60x^4 + 80x^3$