CALCULUS 1

UNIT 1

LIMITS

Lesson 2

LESSON 2

There are various methods that can be used to find a limit. In this lesson we will focus on two methods: a) by direct substitution, and b) algebraically.

FINDING LIMITS BY SUBSTITUTION

Finding limits by substitution is as simple as substituting the number that ***x*** approaches into the function. This is in fact simple algebraic substitution.

Example 1. Solve by substitution.

$$\lim_{x\to 2} x^3 - x^2 - 4 = 2^3 - 2^2 - 4 = 8 - 4 - 4 = 0$$

Example 2. Solve by substitution.

$$\lim_{x\to 1} -\frac{x-4}{x^2-6x+8} = -\frac{1-4}{1^2-6(1)+8} = -\frac{-3}{3} = 1$$

Example 3. Solve by substitution.

In the example below, if we substitute, we will have a zero in the denominator.

To avoid this, we must first factor the expression then cancel out the common factors. Only then can the substitution be done.

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2}$$

$$\lim_{x\to 2} x+2 = 2+2 = 4$$

Example 4. Solve by substitution.

$$\lim_{x\to 1}\left(-\frac{x^2}{2}+2x+4\right) = \left(-\frac{1^2}{2}+2(1)+4\right) = -\frac{1}{2}+2+4 = 5\frac{1}{2}$$

Example 5. Solve by substitution.

$$\lim_{x\to -1} \frac{x^2-3x-4}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-4)}{x+1}$$

$$\lim_{x\to -1} x-4 = -1-4 = -5$$

Lesson 2 Exercise

Determine the limit by Substitution.

Note that in some cases you may have to multiply the numerator and denominator by the conjugate of the denominator.

denominator.

1. $\lim_{x\to 0}\left(x^2-5\right)$ 2. $\lim_{x\to 2}\left(x^3+5x^2-7x+1\right)$ 3. $\lim_{x\to 0}\frac{x^3-6x-8}{x-2}$

4. $\lim_{x \to 8} \frac{x^2+64}{x+8}$

5. $\lim_{x \to 5} \sqrt{x^2+14x+49}$

6. $\lim_{x \to 2} \frac{x^3-5x^2+2x-4}{x^2-3x+3}$

7. $\lim_{x \to 4} \frac{x-\sqrt{x}}{4+\sqrt{x}}$

8. $\lim_{x \to 3} \frac{\sqrt{x^2+7}-3}{x+3}$

9. $\lim_{x \to 0} \frac{x^2-25}{x^2-4x-5}$

10. $\lim_{x \to 1} \left(x^2-4x\right)^3$

FINDING LIMITS ALGEBRAICALLY

Sometime when we substitute in an expression we obtain an **indeterminate** answer, i.e., $\frac{0}{0}$. This means that more math must be done to see if the limit exists. Therefore, finding limits algebraically involves any one of the following:

1. Factoring, then canceling out the common factors.
2. Rationalizing the numerator/denominator.
3. Finding the lowest common denominator (lcd).

Example 1. Solve algebraically.

$$\lim_{x \to 0} \frac{10x}{20x^2+15x}$$

$$\lim_{x \to 0} \frac{5x(2)}{5x(4x+3)} = \lim_{x \to 0} \frac{2}{4x-3} = \frac{2}{0+3} = \frac{2}{3}$$

Example 2. Solve algebraically.

$$\lim_{x \to -2} \frac{x^3+2x^2-36x-72}{x^2-4x-12}$$

$$\lim_{x \to -2} \frac{x^2(x+2)-36(x+2)}{(x-6)(x+2)} = \lim_{x \to -2} \frac{(x^2-36)(x+2)}{(x-6)(x+2)}$$

$$\lim_{x \to -2} \frac{(x-6)(x+6)(x+2)}{(x-6)(x+2)} = \lim_{x \to -2} x+6 = -2+6 = 4$$

Example 3. Solve algebraically.

$$\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$$

$$\lim_{x \to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}-1}$$ Note: $(\sqrt{x} - 1)(\sqrt{x} + 1) = x - 1$.

$$\lim_{x \to 1} \sqrt{x}+1 = 1+1 = 2$$

Example 4. Solve algebraically.

$$\lim_{x\to 6}\frac{2x-12}{\sqrt{x-2}-\sqrt{10-x}}$$

$$\lim_{x\to 6}\frac{2x-12}{\sqrt{x-2}-\sqrt{10-x}}*\frac{\sqrt{x-2}+\sqrt{10-x}}{\sqrt{x-2}+\sqrt{10-x}}=\lim_{x\to 6}\frac{(2x+12)\left(\sqrt{x-2}+\sqrt{10-x}\right)}{2x-12}$$

$$\lim_{x\to 6}\left(\sqrt{x-2}+\sqrt{10-x}\right)=\sqrt{6-2}+\sqrt{10-6}=2+2=4$$

Example 5. Solve algebraically.

$$\lim_{x\to 0}\frac{\frac{1}{x+6}-\frac{1}{6}}{x}$$

$$\lim_{x\to 0}\frac{\frac{1}{x+6}-\frac{1}{6}}{x}=\lim_{x\to 0}\frac{\frac{6-(x+6)}{6(x+6)}}{x}=\lim_{x\to 0}\frac{\frac{6-x-6}{6(x+6)}}{x}$$

$$\lim_{x\to 0}\frac{-x}{6(x+6)}*\frac{1}{x}=\lim_{x\to 0}\frac{-1}{6x+36}=\frac{-1}{36}$$

Lesson 2 Exercise

Determine the limit algebraically if it exists.

1. $\lim_{x\to 2}\frac{x^2+x-6}{x-2}$
2. $\lim_{x\to -4}\frac{x^2+5x+4}{x^2+3x-4}$
3. $\lim_{x\to 2}\frac{x^2-x+6}{x-2}$
4. $\lim_{x\to 4}\frac{x^2-4x}{x^2-3x-4}$
5. $\lim_{x\to -3}\frac{x^2-9}{2x^2+7x+3}$
6. $\lim_{x\to -1}\frac{x^2-4x}{x^2-3x-4}$
7. $\lim_{x\to 4}\frac{x^2-x-12}{x-4}$
8. $\lim_{x\to 1}\frac{x^4+3x^3-13x^2-27x+36}{x^2+3x-4}$
9. $\lim_{x\to 3}\frac{x^2-6x+9}{x^2-2x-3}$
10. $\lim_{x\to 3}\frac{x^2-5x+6}{(x-3)^2}$

Lesson 2 Solutions

Determine the limit algebraically if it exists.

1. $\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}\frac{(x+3)(x-2)}{x-2}=\lim_{x\to 2}x+3=2+3=5$
2. $\lim_{x\to -4}\frac{x^2+5x+4}{x^2+3x-4}=\lim_{x\to -4}\frac{(x+4)(x+1)}{(x+4)(x-1)}=\lim_{x\to -4}\frac{x+1}{x-1}=\frac{-4+1}{-4-1}=\frac{3}{5}$
3. $\lim_{x\to 2}\frac{x^2-x+6}{x-2}$ DNE. Note: $x^2-x+6\to 8,$ while $x-2\to 0.$
4. $\lim_{x\to 4}\frac{x^2-4x}{x^2-3x-4}=\lim_{x\to 4}\frac{x(x-4)}{(x+1)(x-4)}=\lim_{x\to 4}\frac{x}{x+1}=\frac{4}{4+1}=\frac{4}{5}$

5. $\lim_{x\to -3}\frac{x^2-9}{2x^2+7x+3}=\lim_{x\to -3}\frac{(x-3)(x+3)}{(2x+1)(x+3)}=\lim_{x\to -3}\frac{x-3}{2x+1}=\frac{-3-3}{-6+1}=\frac{6}{5}$

6. $\lim_{x\to -1}\frac{x^2-4x}{x^2-3x-4}=\lim_{x\to -1}\frac{x(x-4)}{(x+1)(x-4)}=\lim_{x\to -1}\frac{x}{x+1}=\frac{-1}{-1+1}=\frac{-1}{0}$ DNE

7. $\lim_{x\to 4}\frac{x^2-x-12}{x-4}=\lim_{x\to 4}\frac{(x-4)(x+3)}{x-4}=\lim_{x\to 4}x+3=4+3=7$

8. $\lim_{x\to 1}\frac{x^4+3x^3-13x^2-27x+36}{x^2+3x-4}=\lim_{x\to 1}\frac{(x^2+3x-4)(x^2-9)}{x^2+3x-4}$ (divide numerator by the denominator)

$\lim_{x\to 1}x^2-9=1-9=-8$

9. $\lim_{x\to 3}\frac{x^2-6x+9}{x^2-2x-3}=\lim_{x\to 3}\frac{(x-3)(x-3)}{(x-3)(x+1)}$

$\lim_{x\to 3}\frac{(x-3)}{(x+1)}=\frac{3-3}{3+1}=\frac{0}{4}=0$

10. $\lim_{x\to 3}\frac{x^2-5x+6}{(x-3)^2}=\lim_{x\to 3}\frac{(x-2)(x-3)}{(x-3)(x-3)}=\lim_{x\to 3}\frac{x-2}{x-3}=\frac{3-2}{3-3}=\frac{1}{0}$ DNE