CALCULUS 1

DIFFERENTIATION: USING THE CHAIN RULE

WORKED EXAMPLES

Note: if *f* and *g* are differentiable functions, then $\frac{d}{dx} f\left(g\left(x\right) = f'\left(g\left(x\right) * g'\left(x\right)$ **or in other words, let *y* = *f(u)* and *u* = *g(x)* then** $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$**.**

1) $f\left(x\right) = \left(x+1\right)^{99}$

2) $f\left(x\right) = \sqrt{1-x^2}$

3) $w = \left(t^2+1\right)^{100}$

4) $w = \left(t^3+1\right)^{100}$

5) $w = \left(\sqrt{t}+1\right)^{100}$

6) $f\left(t\right) = e^{3t}$

7) $h\left(w\right) = \left(w^4-2w\right)^5$

8) $w\left(r\right) = \sqrt{r^4+1}$

9) $g\left(x\right) = e^{\pi x}$

10) $f\left(\theta\right) = 2^{-\theta}$

11) $y = \pi^{\left(x+2\right)}$

12) $g\left(x\right) = 3^{\left(2x+7\right)}$

13) $h\left(x\right) = \left(x^3+e^x\right)^4$

14) $f\left(x\right) = e^{2x}\left(x^2+5^x\right)$

15) $v\left(t\right) = t^2 e^{-ct}$

16) $p\left(t\right) = e^{4t+2}$

17) $g\left(t\right) = e^{\left(1+3t\right)^2}$

18) $z\left(x\right) = \sqrt[3]{2^x+5}$

19) $z = 2^{5t-3}$

20) $w = \sqrt{\left(x^2 * 5^x\right)^3}$

21) $y = e^{\frac{3w}{2}}$

22) $y = e^{-4t}$

23) $y = \sqrt{s^3+1}$

24) $w = e^{\sqrt{s}}$

25) $y = te^{-t^2}$

26) $f\left(z\right) = \sqrt{z}e^{-z}$

27) $z = \frac{\sqrt{z}}{e^z}$

28) $y = \frac{\sqrt{z}}{2^z}$

29) $f\left(t\right) = te^{5-2t}$

30) $y = \left(\frac{x^2+2}{3}\right)^2$

31) $h\left(x\right) = \sqrt{\frac{x^2+9}{x+3}}$

32) $y = \frac{e^{2x}}{x^2+1}$

33) $y = \frac{1}{e^{3x}+x^2}$

34) $h\left(z\right) = \left(\frac{b}{a+z^2}\right)^4$

35) $h\left(x\right) = 2^{e^{3x}}$

36) $f\left(z\right) = \frac{1}{\left(e^z+1\right)^2}$

37) $f\left(\theta\right) = \frac{1}{1+e^{-\theta}}$

38) $f\left(x\right) = 6e^{5x}+e^{-x^2}$

39) $f\left(w\right) = \left(5w^2+3\right)e^{w^2}$

40) $w = \left(t^2+3t\right)\left(1-e^{-2t}\right)$

SOLUTIONS

1) $f(x)=(x+1)^{99}$

$f'(x)=99(x+1)^{98}$

2) $f(x)=\sqrt{1-x^2} \Rightarrow (1-x^2)^{\frac{1}{2}}$

$f'(x)=\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$

$=\frac{-2x}{2\sqrt{1-x^2}}=\frac{-x}{\sqrt{1-x^2}}$

3) $w=(t^2+1)^{100}$

$w'=100(t^2+1)^{99}(2t)=200t(t^2+1)^{99}$

4) $w=(t^3+1)^{100}$

$w'=100(t^3+1)^{99}(3t^2)=300t^2(t^3+1)$

5) $w=(\sqrt{t}+1)^{100} \Rightarrow \left(t^{\frac{1}{2}}+1\right)^{100}$

$w'=\frac{1}{2}t^{-\frac{1}{2}}100\left(t^{\frac{1}{2}}+1\right)^{99}=\frac{50(\sqrt{t}+1)^{99}}{\sqrt{t}}$

6) $f(t)=e^{3t}$

$f'(t)=3e^{3t}$

7) $h(w)=(w^4-2w)^5$

$h'(w)=5(w^4-2w)^4(4w^3-2)$

8) $w(r)=\sqrt{r^4+1} \Rightarrow (r^4+1)^{\frac{1}{2}}$

$w'(r)=\frac{1}{2}(r^4+1)^{-\frac{1}{2}}(4r^3)$

$=2\mathrm{r}^3(r^4+1)^{-\frac{1}{2}}=\frac{2r^3}{(r^4+1)^{\frac{1}{2}}}$

9) $g(x)=e^{\pi x}$

$g'(x)=\pi e^{\pi x}$

10) $f(\theta)=2^{-\theta} \Rightarrow (2^{-1})^{\theta} \Rightarrow \left(\frac{1}{2}\right)^{\theta}$

$f'(\theta)=\ln\frac{1}{2}(2^{-\theta})$

11) $y=\pi^{(x+2)}$

$y'=\ln\pi(\pi^{x+2})$

12) $g(x)=3^{(2x+7)}$

$g'(x)=2\ln 3\left(3^{(2x+7)}\right)$

13) $h(x)=(x^3+e^x)^4$

$h'(x)=4(x^3+e^x)^3(3x^2+e^x)$

14) $f(x)=e^{2x}(x^2+5^x)$

$f'(x)=2e^{2x}(x^2+5^x)+(e^{2x})(2x+5^x\ln 5)$

$=e^{2x}\left\{2(x^2+5^x)+(2x+5^x\ln 5)\right\}$

15) $v(t)=t^2e^{-ct}$

$v'(t)=2te^{-ct}-t^2ce^{-ct}$

$=e^{-ct}(2t-ct^2)$

16) $p(t)=e^{4t+2}$

$p'(t)=4e^{4t+2}$

17) $g(t)=e^{(1+3t)^2}$

$$g'(t)=2(1+3t)3e^{(1+3t)^2}$$
$$=6(1+3t)e^{(1+3t)^2}$$

18) $z(x)=\sqrt[3]{2^x+5}\Rightarrow(2^x+5)^{\frac{1}{3}}$

$$z'(x)=\frac{1}{3}(2^x+5)^{-\frac{2}{3}}\ln 2(2^x)$$
$$=\frac{2^x\ln 2}{3\sqrt[3]{(2^x+5^x)^2}}$$

19) $z=2^{5t-3}$

$$z'=5\ln 2(2^{5t-3})$$

20) $w=\sqrt{(x^2*5^x)^3}\Rightarrow(x^2*5^x)^{\frac{3}{2}}$

$$w'=\frac{3}{2}(x^2*5^x)^{\frac{1}{2}}(2x*5^x+5^x\ln 5*x^2)$$
$$=\frac{3}{2}(x^2*5^x)^{\frac{1}{2}}(x*5^x)(2+x\ln 5)$$
$$=\frac{3}{2}x^2\sqrt{5^{3x}}(2+x\ln 5)$$

21) $y=e^{\frac{3w}{2}}$

$$y'=\frac{3}{2}e^{\frac{3w}{2}}$$

22) $y=e^{-4t}$

$$y'=-4e^{-4t}$$

23) $y=\sqrt{s^3+1}\Rightarrow(s^3+1)^{\frac{1}{2}}$

$$y'=\frac{1}{2}(s^3+1)^{-\frac{1}{2}}(3s^2)$$
$$=\frac{3s}{2\sqrt{s^3+1}}$$

24) $w=e^{\sqrt{s}}=e^{s^{\frac{1}{2}}}$

$$w'=\frac{1}{2}s^{-\frac{1}{2}}e^{s^{\frac{1}{2}}}\Rightarrow\frac{e^{\sqrt{s}}}{2\sqrt{s}}$$

25) $y=te^{-t^2}$

$$y'=(1)(e^{-t^2})+te^{-t^2}(-2t)$$

26) $f(z)=\sqrt{z}e^{-z}\Rightarrow z^{\frac{1}{2}}e^{-z}$

$$f'(z)=\frac{1}{2}z^{-\frac{1}{2}}e^{-z}+\left(z^{\frac{1}{2}}*-e^{-z}\right)$$
$$=\frac{e^{-z}}{2\sqrt{z}}-\sqrt{z}e^{-z}$$

27) $z=\frac{\sqrt{z}}{e^z}\Rightarrow\frac{z^{\frac{1}{2}}}{e^z}$

$$z'=\frac{\frac{e^z}{2\sqrt{z}}-\sqrt{z}e^z}{(e^z)^2}$$

28) $y=\frac{\sqrt{z}}{2^z}\Rightarrow\frac{z^{\frac{1}{2}}}{2^z}$

$$y' = \frac{2^z\left(\frac{1}{2}z^{-\frac{1}{2}}\right) - z^{\frac{1}{2}}\left(2^z\right)\ln 2}{2^{2z}}$$

$$= \frac{2^z\left\{\frac{1}{2\sqrt{z}} - \sqrt{z}\ln 2\right\}}{2^{2z}} \quad \left(\text{factor out } 2^z\right)$$

$$= \frac{\frac{1}{2\sqrt{z}} - \sqrt{z}\ln 2}{2^z} \quad \left(\text{cancel } 2^z\right)$$

$$= \frac{\frac{1-2z\ln 2}{2\sqrt{z}}}{2^z} \quad \left(\text{common denominator}\right)$$

$$= \frac{1-2z\ln 2}{2^z * 2\sqrt{z}} \quad \left(\text{multiply by } 2\sqrt{z}\right)$$

$$= \frac{1-2z\ln 2}{2^{z+1} * \sqrt{z}}$$

29) $f(t) = te^{5-2t}$

$$f'(t) = (1)\left(e^{5-2t}\right) + t\left(-2e^{5-2t}\right)$$

$$= e^{5-2t}(1-2t)$$

30) $y = \left(\frac{x^2+2}{3}\right)^2 \Rightarrow \frac{1}{9}\left(x^2+2\right)^2$

$$y' = \frac{1}{9} * 2\left(x^2+2\right)2x$$

$$= \frac{4x}{9}\left(x^2+2\right)$$

31) $h(x) = \sqrt{\frac{x^2+9}{x+3}} \Rightarrow \left(\frac{x^2+9}{x+3}\right)^{\frac{1}{2}}$

$$h'(x) = \frac{1}{2}\left(\frac{x^2+9}{x\ +3}\right)^{-\frac{1}{2}}\left(\frac{2x(x+3)-(1)\left(x^2+9\right)}{(x+3)^2}\right)$$

$$= \frac{1}{2}\left(\frac{x^2+9}{x+3}\right)^{-\frac{1}{2}}\left(\frac{2x^2+6x-x^2-9}{(x+3)^2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{x+3}{x^2+9}}\left(\frac{x^2+6x-9}{(x+3)^2}\right)$$

32) $y = \frac{e^{2x}}{x^2+1}$

$$y' = \frac{2e^{2x}\left(x^2+1\right) - 2xe^{2x}}{\left(x^2+1\right)^2}$$

$$= \frac{2e^{2x}\left(x^2+1-x\right)}{\left(x^2+1\right)^2}$$

$$= \frac{2e^{2x}\left(x^2-x+1\right)}{\left(x^2+1\right)^2}$$

33) $y = \frac{1}{e^{3x}+x^2} \Rightarrow \left(e^{3x}+x^2\right)^{-1}$

$$y' = -1\left(e^{3x}+x^2\right)^{-2}\left(3e^{3x}+2x\right)$$

$$= -\frac{3e^{3x}+2x}{\left(e^{3x}+x^2\right)^2}$$

34) $h(z) = \left(\frac{b}{a+z^2}\right)^4$

$$h'(z) = 4\left(\frac{b}{a+z^2}\right)^3\left(\frac{(0)\left(a+z^2\right)-2bz}{\left(a+z^2\right)^2}\right)$$

$$= \frac{-8b^4z}{\left(a+z^2\right)^5}$$

35) $h(x) = 2^{e^{3x}}$

$$h'(x) = \left(2^{e^{3x}}\right)\left(3e^{3x}\right)\ln 2$$

36) $f(z) = \frac{1}{\left(e^z+1\right)^2} \Rightarrow \left(e^z+1\right)^{-2}$

$$f'(z) = -2\left(e^z+1\right)^{-3}\left(e^z\right)$$

$$= \frac{-2e^z}{\left(e^z+1\right)^3}$$

37) $f(\theta) = \frac{1}{1+e^{-\theta}}$

$$f'(\theta) = \frac{(0)\left(1+e^{-\theta}\right)-e^{-\theta}}{\left(1+e^{-\theta}\right)^2}$$

$$= -\frac{e^{-\theta}}{\left(1+e^{-\theta}\right)^2}$$

38) $f(x) = 6e^{5x} + e^{-x^2}$

$f'(x) = 30e^{5x} - 2xe^{-x^2}$

39) $f(w) = \left(5w^2+3\right)e^{w^2}$

$$f'(w) = \left(e^{w^2}\right)10w + \left(5w^2+3\right)\left(2we^{w^2}\right)$$

$$= 10we^{w^2} + \left(5w^2+3\right)\left(2we^{w^2}\right)$$

$$= 2we^{w^2}\left(5+5w^2+3\right)$$

$$= 2we^{w^2}\left(5w^2+8\right)$$

40) $w = \left(t^2+3t\right)\left(1-e^{-2t}\right)$

$w' = (2t+3)\left(1-e^{-2t}\right) + \left(t^2+3t\right)\left(2e^{-2t}\right)$