CALCULUS 1

TANGENT TO A CURVE - LIMITS

WORKED EXAMPLES

NOTE:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

Therefore, $f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

Use limits to find the slope of the following.

1. $f(x) = -5x - 6$
2. $f(x) = 7x - 4$
3. $f(x) = 3x - 8$
4. $f(x) = 2x^2 - 5x + 3$
5. $f(x) = 3x^3 - 2x^2 + 5$

Find the slope of a line at a given point.

6. $y = x^2 + 3$, at (2, 5)
7. $y = 4x^2 - 2x + 1$, at $(1, 3)$
8. $y = 2x^2 + x - 3$, at$(3, 18)$
9. $y = x^3 + 2x^2$, at(2,16)
10. $y = 2x^3 - x^2 - 1$, at$(2, 11)$

SOLUTIONS.

1. $f(x) = -5x - 6$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-5(x+h^2) - 6 - (-5x - 6)}{h}$$

$$\lim_{h \to 0} \frac{-5x - 5h - 6 + 5x + 6}{h} = \lim_{h \to 0} \frac{-5h}{h} = -5.$$

2. $f(x) = 7x - 4$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{7(x+h) - f(7x - 4)}{x}$$

$$\lim_{h \to 0} \frac{7x + 7h - 4 - 7x + 4}{h} = \lim_{h \to 0} \frac{7h}{h} = 7$$

3. $f(x) = 3x - 8$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 8 - (3x - 8)}{h}$$

$$\lim_{h \to 0} \frac{3x + 3h - 8 - 3x + 8}{h} = \lim_{h \to 0} \frac{3h}{h} = 3$$

4. $f\left(x\right)=2x^2-5x+3$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2(x+h)^2-5(x+h)+3-(2x^2-5x+3)}{h}$$

$$\lim_{h\to 0}\frac{2(x^2+2xh+2h^2)-5(x+h)+3-(2x^2-5x+3)}{h}$$

$$\lim_{h\to 0}\frac{2x^2+4xh+2h^2-5x-5h+3-2x^2+5x-3}{h}$$

$$\lim_{h\to 0}\frac{4xh+2h^2-5h}{h}=\lim_{h\to 0}4x+h-5=4x-5$$

5. $f\left(x\right)=3x^3-2x^2+5$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{3(x+h)^3-2(x+h)^2+5-(3x^3-2x^2+5)}{h}$$

$$\lim_{h\to 0}\frac{3(x^3+3x^2h+3xh^2+h^3)-2(x^2+2xh+h^2)+5-(3x^3-2x^2+5)}{h}$$

$$\lim_{h\to 0}\frac{3x^3+9x^2h+9xh^2+3h^3-2x^2-4xh+2h^2+5-3x^3+2x^2-5}{h}$$

$$\lim_{h\to 0}\frac{9x^2h+9xh^2+3h^3-4xh+2h^2}{h}=\lim_{h\to 0}9x^2+9xh+3h^2-4x+2h=9x^2-4x.$$

6. $y=x^2+3,$ at (2, 5)

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+3-(x^2+3)}{h}$$

$$\lim_{h\to 0}\frac{x^2+2xh+h^2+3-x^2-3}{h}-=\lim_{h\to 0}\frac{2xh+h^2}{h}$$

$$\lim_{h\to 0}2x+h=2x$$

The slope of the line is 2*x*. Therefore, the slope at *x* = 2 is 2^2+3 = 7.
The equation of the line is *y* – 7 = 7 (*x* – 2) or *y* = 7*x* – 7.

7. $y=4x^2-2x+1$, at $(1,3)$

$$\lim_{h\to 0}\frac{f(x-h)-f(x)}{h}=\lim_{h\to 0}\frac{4(x+h)^2-2(x+h)+1-(4x^2-2x+1)}{h}$$

$$\lim_{h\to 0}\frac{4(x^2+2xh+h^2)-2(x+h)+1-(4x^2-2x+1)}{h}$$

$$\lim_{h\to 0}\frac{4x^2+8xh+4h^2-2x-2h+1-4x^2+2x-1}{h}$$

$$\lim_{h\to 0}\frac{8xh+4h^2-2h}{h}=\lim_{h\to 0}8x+4h-2=8x-2$$

The slope of the line is 8*x* – 2. Therefore, the slope at *x* = 1is 8(1) – 2 = 6.
The equation of the line is *y* – 3 = 6 (*x* – 1) or *y* = 6*x* – 3.

8. $y=2x^2+x-3$, at$(3,18)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) - 3 - (2x^2 + x - 3)}{h}$$

$$\lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + (x+h) - 3 - (2x^2 + x - 3)}{h}$$

$$\lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + x + h - 3 - 2x^2 - x + 3}{h}$$

$$\lim_{h \to 0} \frac{4xh + 2h^2 + h}{h} = \lim_{h \to 0} 4x + 2h + 1 = 4x + 1$$

The slope of the line is $4x + 1$. Therefore, the slope at $x = 3$ is $4(3) + 1 = 13$
The equation of the line is $y - 18 = 13(x - 3)$ or $y = 13x - 21$.

9. $y = x^3 + 2x^2$, at$(2, 16)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 + 2(x+h)^2 - (x^3 + 2x^2)}{h}$$

$$\lim_{h \to 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) + 2(x^2 + 2xh + h^2) - (x^3 + 2x^2)}{h}$$

$$\lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x^2 + 4xh + 2h^2 - x^3 - 2x^2}{h}$$

$$\lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 4xh + 2h^2}{h} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2 + 4x + 2h}{h}$$

$$= 3x^2 + 4x.$$

The slope of the line is $3x^2 + 4x$. Therefore, the slope at $x = 2$ is $3(2)^2 + 4(2) = 20$
The equation of the line is $y - 16 = 20(x - 2)$ or $y = 20x - 24$.

10. $y = 2x^3 - x^2 - 1$, at$(2, 11)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h \to 0} \frac{2(x+h)^3 - (x+h)^2 - 1 - (2x^3 - x^2 - 1)}{h}$$

$$\lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - (x^2 + 2xh + h^2) - 1 - (2x^3 - x^2 - 1)}{h}$$

$$\lim_{h \to 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - x^2 - 2xh - h^2 - 1 - 2x^3 + x + 1}{h}$$

$$\lim_{h \to 0} \frac{6x^2 + 6xh + 2h^2 - 2x - h}{h} = 6x^2 - 2x.$$

The slope of the line is $6x^2 - 2x$. Therefore, the slope at $x = 2$ is $6(2)^2 - 2(2) = 20$
The equation of the line is $y - 11 = 20(x - 2)$ or $y = 20x - 29$.