CALCULUS 2

L'HOPITALS RULE

FINDING LIMITS USING L'HOSPITAL'S RULE

Use L'Hospital's Rule to find the limits of the following.

1) $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}$

2) $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x-3}$

3) $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}-2}$

4) $\lim\limits_{x\to 0}\dfrac{x}{1-e^x}$

5) $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^3-x^2-x+1}$

6) $\lim\limits_{x\to 1}\dfrac{1-x+\ln x}{x^3-3x+2}$

7) $\lim\limits_{x\to 0}\dfrac{x\cos x-\sin x}{x}$

8) $\lim\limits_{x\to \infty}\dfrac{x^2-1}{4x^2+2}$

9) $\lim\limits_{x\to \infty}\dfrac{2x^2+4x-7}{x^3+3x^2-5}$

10) $\lim\limits_{x\to \infty}\dfrac{x^3}{e^x}$

11) $\lim\limits_{x\to 1}\dfrac{5\ln x}{x-1}$

12) $\lim\limits_{x\to 0}\dfrac{3x}{\ln(x+1)}$

13) $\lim\limits_{x\to 0^+}5x^2\ln x$

14) $\lim\limits_{x\to \frac{\pi}{2}}(\sec x-\tan x)$

15) $\lim\limits_{x\to \left(\frac{\pi}{2}\right)^+}\dfrac{\cos x}{1-\sin x}$

16) $\lim\limits_{x\to 0}\dfrac{x+\tan x}{\sin x}$

17) $\lim\limits_{x\to 0}\dfrac{e^x-1-x}{x^2}$

18) $\lim\limits_{x\to \infty}\dfrac{5x+e^{-x}}{7x}$

19) $\lim\limits_{x\to \infty}xe^{-x}$

20) $\lim\limits_{x\to \infty}4xe^{-x}$

21) $\lim\limits_{x\to 0^+}\sqrt{x}\ln x$

22) $\lim\limits_{x\to 0}\cot 2x\sin 6x$

23) $\lim\limits_{x\to \infty}x^3e^{-x^2}$

24) $\lim\limits_{x\to 1^+}\ln x\tan\left(\dfrac{\pi x}{2}\right)$

25) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\csc x\right)$

Solutions

1) $\lim\limits_{x\to -1}\dfrac{2x^2-x-3}{x+1}=\dfrac{0}{0}\to \text{L'H}\ \lim\limits_{x\to -1}\dfrac{4x-1}{1}=\dfrac{-5}{1}=-5$

2) $\lim\limits_{x\to 3}\dfrac{x^2-x-6}{x-3}=\dfrac{0}{0}\to \text{L'H}\ \lim\limits_{x\to 3}\dfrac{2x-1}{1}=5$

3) $\lim\limits_{x\to 1}\dfrac{x-1}{\sqrt{x^2+3}-2}=\dfrac{0}{0}\to \lim\limits_{x\to 1}\dfrac{x-1}{\left(x^2+3\right)^{\frac{1}{2}}-2}$

$$\rightarrow \text{L'H} \lim_{x\to 1} \frac{1}{\frac{1}{2}\left(x^2+3\right)^{-\frac{1}{2}}\left(2x\right)} \rightarrow \lim_{x\to 1} \frac{1}{x\left(x^2+3\right)^{-\frac{1}{2}}} \rightarrow \lim_{x\to 1} \frac{\left(x^2+3\right)^{\frac{1}{2}}}{x} = \frac{(1+3)^{\frac{1}{2}}}{1} = 2$$

4) $\lim_{x\to 0} \frac{x}{1-e^x} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to 0} \frac{1}{-e^x} = \frac{1}{-1} = -1$

5) $\lim_{x\to 1} \frac{x^3-3x+2}{x^3-x^2-x+1} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to 1} \frac{3x^2-3}{3x^2-2x-1} = \frac{0}{0}$

$\rightarrow \text{L'H} \lim_{x\to 1} \frac{6x}{6x-2} = \frac{6}{4} = \frac{3}{2}$

6) $\lim_{x\to 1} \frac{1-x+\ln x}{x^3-3x+2} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to 1} \frac{-1+\frac{1}{x}}{3x^2-3} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to 1} \frac{-\frac{1}{x^2}}{6x} = -\frac{1}{6}$

7) $\lim_{x\to 0} \frac{x\cos x - \sin x}{x} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to 0} \frac{\cos x - x\sin x - \cos x}{1} = 1-0-1 = 0$

8) $\lim_{x\to \infty} \frac{x^2-1}{4x^2+2} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{2x}{8x} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{2}{8} = \frac{1}{4}$

9) $\lim_{x\to \infty} \frac{2x^2+4x-7}{x^3+3x^2-5} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{4x+4}{3x^2+6x} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{4}{6x+6} = \frac{4}{\infty} = 0$

10) $\lim_{x\to \infty} \frac{x^3}{e^x} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{3x^2}{e^x} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{6x}{e^x} = \frac{\infty}{\infty} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{6}{e^x} = \frac{6}{\infty} = 0$

11) $\lim_{x\to 1} \frac{5\ln x}{x-1} = \frac{0}{0} \rightarrow \text{L'H } 5\lim_{x\to 1} \frac{\frac{1}{x}}{1} = \frac{\frac{5}{x}}{1} = 5$

12) $\lim_{x\to 0} \frac{3x}{\ln\left(x+1\right)} = \frac{0}{0} \rightarrow \text{L'H} \lim_{x\to \infty} \frac{3}{\frac{1}{x+1}} = \frac{3}{\frac{1}{1}} = 3$

13) $\lim_{x\to 0^+} 5x^2 \ln x = 0 * -\infty \rightarrow 5\lim_{x\to 0} \frac{\ln x}{\frac{1}{x^{-2}}} = \frac{-\infty}{\infty}$

$$\to \text{L'H } 5\lim_{x\to 0}\frac{\frac{1}{x}}{\frac{1}{-2x^{-3}}} \to 5\lim_{x\to 0}\frac{1}{x}*\frac{x^3}{-2}=\frac{5x^2}{-2}=\frac{0}{-2}=0$$

14) $$\lim_{x\to\frac{\pi}{2}}\left(\sec x-\tan x\right)=\infty-\infty \to \lim_{x\to\frac{\pi}{2}}\frac{1}{\cos x}-\frac{\sin x}{\cos x}\to \lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}=\frac{1-1}{0}=\frac{0}{0}$$

$$\to \text{ L'H } \lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{-\sin x}=\frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}=\frac{0}{1}=0$$

15) $$\lim_{x\to\left(\frac{\pi}{2}\right)^+}\frac{\cos x}{1-\sin x}=\frac{0}{1-1}=\frac{0}{0}\to \text{L'H } \lim_{x\to\left(\frac{\pi}{2}\right)^+}\frac{-\sin x}{-\cos x}=\tan x=-\infty$$

16) $$\lim_{x\to 0}\frac{x+\tan x}{\sin x}=\frac{0}{0}\to \text{L'H } \lim_{x\to 0}\frac{1+\sec^2 x}{\cos x}=\frac{1+1}{1}=2$$

17) $$\lim_{x\to 0}\frac{e^x-1-x}{x^2}=\frac{0}{0}\to \text{L'H } \lim_{x\to 0}\frac{e^x-1}{2x}\to \lim_{x\to 0}\frac{0}{0}\to \text{L'H } \lim_{x\to 0}\frac{e^x}{2}=\frac{1}{2}$$

18) $$\lim_{x\to\infty}\frac{5x+e^{-x}}{7x}=\frac{\infty}{\infty}\to \text{L'H } \lim_{x\to\infty}\frac{5-e^{-x}}{7}\to \lim_{x\to\infty}\frac{5-0}{7}=\frac{5}{7}$$

19) $$\lim_{x\to\infty}xe^{-x}=\infty*0\to \lim_{x\to\infty}\frac{x}{e^x}=\frac{\infty}{\infty}\to \text{L'H } \lim_{x\to\infty}\frac{1}{e^x}=\frac{1}{\infty}=0$$

20) $$\lim_{x\to\infty}4xe^{-x}=\infty*0\to \lim_{x\to\infty}\frac{4x}{e^x}=\frac{\infty}{\infty}\to \text{L'H } 4\lim_{x\to\infty}\frac{1}{e^x}=\frac{4*1}{\infty}=0$$

21) $$\lim_{x\to 0^+}\sqrt{x}\ln x=0*-\infty\to \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x^{-\frac{1}{2}}}}=\frac{-\infty}{\infty}\to \text{L'H } \lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{1}{-\frac{1}{2}x^{-\frac{3}{2}}}}$$

$$\to \lim_{x\to 0^+}\frac{1}{x}*\frac{x^{\frac{3}{2}}}{-2}\to \lim_{x\to 0^+}\frac{x^{\frac{1}{2}}}{-2}=\frac{0}{-2}=0$$

22) $$\lim_{x\to 0}\cot 2x\sin 6x=\infty*0\to \lim_{x\to 0}\frac{\sin 6x}{\tan 2x}=\frac{0}{0}$$

$$\rightarrow \text{L'H } \lim_{x\to 0}\frac{6\cos 6x}{2\sec^2 2x}=\frac{6*1}{2*1}=\frac{6}{2}=3$$

23) $$\lim_{x\to\infty} x^3 e^{-x^2}=\infty*0\rightarrow \lim_{x\to\infty}\frac{x^3}{e^{x^2}}=\frac{\infty}{\infty}\rightarrow \text{L'H } \lim_{x\to\infty}\frac{3x^2}{2xe^{x^2}}=\frac{\infty}{\infty}$$

$$\rightarrow \lim_{x\to\infty}\frac{3x}{2e^{x^2}}\rightarrow \text{L'H}\lim_{x\to\infty}\frac{3}{4xe^{x^2}}=\frac{3}{\infty}=0$$

24) $$\lim_{x\to 1^+}\ln x\tan\left(\frac{\pi x}{2}\right)=0*-\infty\rightarrow \lim_{x\to 1^+}\frac{\ln x}{\cot\left(\frac{\pi x}{2}\right)}=\frac{0}{0}\rightarrow \text{L'H}\lim_{x\to 1^+}\frac{\frac{1}{x}}{-\frac{\pi}{2}\csc^2\left(\frac{\pi x}{2}\right)}=\frac{1}{\frac{-\pi}{2}*1}=-\frac{2}{\pi}$$

25) $$\lim_{x\to 0}\left(\frac{1}{x}-\csc x\right)=\infty-\infty\rightarrow \lim_{x\to 0}\frac{1}{x}-\frac{1}{\sin x}\rightarrow \lim_{x\to 0}\frac{\sin x-x}{x\sin x}=\frac{0}{0}$$

$$\rightarrow \text{L'H}\lim_{x\to 0}\frac{\cos x-1}{\sin x+x\cos x}=\frac{0}{0}\rightarrow \text{L'H }\lim_{x\to 0}\frac{-\sin x}{\cos x+\cos x-x\sin x}=\frac{0}{2}=0$$