CALCULUS 1
INDEFINITE INTEGRALS
WORKED EXAMPLES

Note: 1. $\int k\,dx = kx + C$ **2.** $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$ **3.** $\int \dfrac{1}{2}dx = \ln|x| + C$

     4. $\int \cos x\,dx = \sin x + C$ **5.** $\int \sin x\,dx = -\cos x + C$

Evaluate the following.

1) $\int x^r dx$

2) $\int (2x^3 - 5x^2 + 3x + 1)dx$

3) $\int \left(5 - \dfrac{1}{\sqrt{x}}\right)dx = \int \left(5 - x^{-\frac{1}{2}}\right)dx$

4) $\int (2\sqrt[4]{x})dx = 2\int x^{\frac{1}{4}}dx$

5) $\int (5\sqrt[3]{x^2})dx = 5\int \left(x^{\frac{2}{3}}\right)dx$

6) $\int \dfrac{3}{x^4}dx = 3\int x^{-4}dx$

7) $\int (x^2 - 1)\sqrt{x}\,dx = \int (x^2 - 1)x^{\frac{1}{2}}dx$

8) $\int \left(\dfrac{1}{x^3} - \dfrac{1}{x^5}\right)dx = \int (x^{-3} - x^{-5})dx$

9) $\int \dfrac{3x^2 - 2x + 1}{\sqrt{x}} = \int 3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

10) $\int 3\sin x + 5\cos x\,dx$

11) $\int 7\sec^2 x - \sec x \tan x\,dx$

12) $\int 7\csc^2 x + 3x^2 dx$

13) $\int x\sqrt{3x}\,dx = \int \left(\sqrt{3x^{\frac{3}{2}}}\right)dx$

14) $\int \dfrac{1}{\sec x}dx = \int \cos x\,dx$

15) $\int \tan^2 x\,dx = \int (\sec^2 x - 1)dx$

16) $\int \sin\theta\,d\theta$

17) $\int (x^3 - 2)dx$

18) $\int \left(t^2 + \dfrac{1}{t^2}\right)dt = \int (t^2 + t^{-2})dt$

19) $\int 4\sqrt{w}\,dw = 4\int w^{\frac{1}{2}}dw$

20) $\int 5e^z dz$

21) $\int \left(x + \dfrac{1}{\sqrt{x}}\right)dx = \int \left(x + x^{-\frac{1}{2}}\right)dx$

22) $\int (\pi + x^{11})dx$

23) $\int \left(t\sqrt{t} + \dfrac{1}{t\sqrt{t}}\right)dt = \int \left(t^{\frac{3}{2}} + t^{-\frac{3}{2}}\right)dt$

24) $\int \cos(x+1)dx$

25) $\int e^{2x}dx$

SOLUTIONS

1. $\int x^r dx = \dfrac{x^{r+1}1}{r+1} + C$

2. $\int \left(2x^3 - 5x^2 + 3x + 1\right)dx$

$= \dfrac{2x^4}{4} - \dfrac{5x^3}{3} + \dfrac{3x^2}{2} + x + C$

$= \dfrac{1}{2}x^4 - \dfrac{5}{3}x^3 + \dfrac{3}{2}x^2 + x + C$

3. $\int \left(5 - \dfrac{1}{\sqrt{x}}\right)dx = \int \left(5 - x^{-\frac{1}{2}}\right)dx$

$= 5x - 2\sqrt{x} + C$

4. $\int \left(2\sqrt[4]{x}\right)dx = 2\int x^{\frac{1}{4}}dx$

$= \dfrac{8}{5}x^5 + C$

5. $\int \left(5\sqrt[3]{x^2}\right)dx = 5\int \left(x^{\frac{2}{3}}\right)dx$

$= \dfrac{15}{5}x^{\frac{5}{3}} = 3x^{\frac{5}{3}} + C$

6. $\int \dfrac{3}{x^4}dx = 3\int x^{-4}dx$

$= 3\dfrac{x^{-3}}{-3} + C = -\dfrac{1}{x^3} + C$

7. $\int \left(x^2 - 1\right)\sqrt{x}dx = \int \left(x^2 - 1\right)x^{\frac{1}{2}}dx$

$= \int \left(x^{\frac{5}{2}} - x^{\frac{1}{2}}\right)dx = \dfrac{2}{7}x^{\frac{7}{2}} - \dfrac{2}{3}x^{\frac{3}{2}} + C$

8. $\int \left(\dfrac{1}{x^3} - \dfrac{1}{x^5}\right)dx = \int \left(x^{-3} - x^{-5}\right)dx$

$= \dfrac{1}{2x^2} - \dfrac{1}{4x^4} + C = \dfrac{1}{2x^2}\left(1 - \dfrac{1}{2x^2}\right) + C$

9. $\int \dfrac{3x^2 - 2x + 1}{\sqrt{x}} = \left(3x^2 - 2x + 1\right)\left(x^{-\frac{1}{2}}\right)$

$= \int 3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$

$= 2*\dfrac{3}{5}x^{\frac{5}{2}} - 2*\dfrac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

$= \dfrac{6}{5}x^{\frac{5}{2}} - \dfrac{4}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

10. $\int 3\sin x + 5\cos x dx$

$= -3\cos x + 5\sin x + C$

11. $\int 7\sec^2 x - \sec x \tan x dx$

$= 7\tan x - \sec x + C$

12. $\int 7\csc^2 x + 3x^2 dx$

$= -\cot x + \dfrac{3x^3}{3} + C = -\cot x + x^3 + C$

13. $\int x\sqrt{3x}dx = \int \left(\sqrt{3x^{\frac{3}{2}}}\right)dx$

$\int \sqrt{3}*x^{\frac{3}{2}} = \dfrac{2\sqrt{3}*x^{\frac{5}{2}}}{5} + C$

14. $\int \dfrac{1}{\sec x}dx = \int \cos x dx = \sin x + C$

15. $\int \tan^2 x dx = \int \left(\sec^2 x - 1\right)dx$

$= \tan x - x + C$

16. $\int \sin\theta d\theta = -\cos\theta + C$

17. $\int \left(x^3 - 2\right)dx = \dfrac{x^4}{4} - 2x + C$

18. $\int \left(t^2 + \dfrac{1}{t^2}\right)dt = \int \left(t^2 + t^{-2}\right)dt$

$$= \frac{t^3}{3} + t^{-1} + C = \frac{t^3}{3} + \frac{1}{t} + C$$

19. $\int 4\sqrt{w}\,dw = 4\int w^{\frac{1}{2}}\,dw$

$$= \frac{2*4w^{\frac{3}{2}}}{3} + C = \frac{8}{3}\sqrt{w^3} + C$$

20. $\int 5e^z\,dx = 5e^z + C$

21. $\int\left(x + \frac{1}{\sqrt{x}}\right)dx = \int\left(x + x^{-\frac{1}{2}}\right)dx$

$$= \frac{x^2}{2} + 2\sqrt{x} + C$$

22. $\int\left(\pi + x^{11}\right)dx = \pi x + \frac{x^{12}}{12} + C$

23. $\int\left(t\sqrt{t} + \frac{1}{t\sqrt{t}}\right)dt = \int\left(t^{\frac{3}{2}} + t^{-\frac{3}{2}}\right)dt$

$$= \frac{2}{5}t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + C$$

24. $\int \cos(x+1)\,dx = \sin(x+1) + C$

25. $\int e^{2x}\,dx = \frac{e^{2x}}{2} + C$

Note: $\left(\frac{e^u}{u'}\right)$