HARDER CHAIN RULE PROBLEMS

WORKED EXAMPLES

Find the derivative using the Chain Rule.

1. $f(x)=\left(4x^5-1\right)\sqrt[3]{x+1}$

Solution.

$$f'(x)=20x^4*\sqrt[3]{x+1}+\left(4x^5-1\right)*\frac{1}{3}(x+1)^{-\frac{2}{3}}$$

$$=20x^4*\sqrt[3]{x+1}+\frac{\left(4x^5-1\right)}{3(x+1)^{\frac{2}{3}}}=\frac{20x^4*(x+1)^{\frac{1}{3}}*3(x+1)^{\frac{2}{3}}+\left(4x^5-1\right)}{3(x+1)^{\frac{2}{3}}}$$

$$=\frac{20x^4*3(x+1)+\left(4x^5-1\right)}{3(x+1)^{\frac{2}{3}}}=\frac{20x^4*(3x+3)+\left(4x^5-1\right)}{3(x+1)^{\frac{2}{3}}}$$

$$=\frac{60x^5+60x^4+4x^5-1}{3(x+1)^{\frac{2}{3}}}=\frac{64x^5+60x^4-1}{3(x+1)^{\frac{2}{3}}}$$

2. $f(x)=\sqrt{-x^4-1}\,(-x-2)$

Solution.

$$f'(x)=\frac{1}{2}\left(-x^4-1\right)^{-\frac{1}{2}}*-4x^3*(-x-2)+\left(-x^4-1\right)^{\frac{1}{2}}(-1)$$

$$=-2x^3\left(-x^4-1\right)^{-\frac{1}{2}}*(-x-2)-\left(-x^4-1\right)^{\frac{1}{2}}=\frac{-2x^3*(-x-2)}{\left(-x^4-1\right)^{\frac{1}{2}}}-\left(-x^4-1\right)^{\frac{1}{2}}$$

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$$=\frac{-2x^3*(-x-2)-\left(-x^4-1\right)^{\frac{1}{2}}*\left(-x^4-1\right)^{\frac{1}{2}}}{\left(-x^4-1\right)^{\frac{1}{2}}}=\frac{2x^4+4x^3+x^4+1}{\left(-x^4-1\right)^{\frac{1}{2}}}=\frac{3x^4+4x^3+1}{\left(-x^4-1\right)^{\frac{1}{2}}}$$

3. $f(x)=\dfrac{\sqrt[5]{x^2-3}}{-x-5}=\dfrac{\left(x^2-3\right)^{\frac{1}{5}}}{-x-5}$

Solution.

$$f'=\frac{\frac{1}{5}\left(x^2-3\right)^{-\frac{4}{5}}(2x)(-x-5)-\left(x^2-3\right)^{\frac{1}{5}}(-1)}{(-x-5)^2}$$

$$=\frac{\dfrac{2x(-x-5)}{5\left(x^2-3\right)^{\frac{4}{5}}}+\dfrac{\left(x^2-3\right)^{\frac{1}{5}}}{1}}{(-x-5)^2}=\frac{2x(-x-5)+5\left(x^2-3\right)^{\frac{4}{5}}\left(x^2-3\right)^{\frac{1}{5}}}{5\left(x^2-3\right)^{\frac{4}{5}}(-x-5)^2}$$

$$=\frac{-2x^2-10x+5\left(x^2-3\right)}{5\left(x^2-3\right)^{\frac{4}{5}}(-x-5)^2}=\frac{-2x^2-10x+5x^2-15}{5\left(x^2-3\right)^{\frac{4}{5}}(-x-5)^2}=\frac{3x^2-10x-15}{5\left(x^2-3\right)^{\frac{4}{5}}(-x-5)^2}$$

4. $f(x)=\left(\dfrac{5x^5-3}{-3x^3+1}\right)^3$

Solution.

$$f'(x)=3\left(\frac{5x^5-3}{-3x^3+1}\right)^2*\frac{(-3x^3+1)*(25x^4)-(5x^5-3)(-9x^2)}{(-3x^3+1)^2}$$

$$=3\left(\frac{5x^5-3}{-3x^3+1}\right)^2*\frac{-75x^7+25x^4+45x^7-27x^2}{(-3x^3+1)^2}$$

$$=\frac{3(5x^5-3)^2(-30x^7+25x^4-27x^2)}{(-3x^3+1)^4}$$

$$=\frac{3x^2(5x^5-3)^2(-30x^5+25x^2-27)}{(-3x^3+1)^4}$$

5. $f(x)=\left(\dfrac{x^5+4}{x^2-5}\right)^{\frac{1}{5}}$

Solution.

$$f'(x)=\left\{\frac{1}{5}\left(\frac{x^5+4}{x^2-5}\right)^{-\frac{4}{5}}(5x^4)\right\}\left(\frac{5x^4(x-5)^{\frac{1}{5}}}{(x^2-5)^{\frac{2}{5}}}-\frac{2x(x^5+4)^{\frac{1}{5}}}{(x^2-5)^{\frac{2}{5}}}\right)$$

$$=\frac{5x^4(x^2-5)^{\frac{1}{5}}(x^5+4)^{-\frac{4}{5}}}{5(x^2-5)^{\frac{2}{5}}}-\frac{2x(x^5+4)^{\frac{1}{5}}(x^2-5)^{-\frac{4}{5}}}{5(x^2-5)^{\frac{2}{5}}}$$

$$=\frac{5x^4(x^2-5)}{5(x^2-5)^{\frac{2}{5}}(x^5+4)^{\frac{4}{5}}}-\frac{2x(x^5+4)}{5(x^2-5)^{\frac{2}{5}}(x^2-5)^{\frac{4}{5}}}$$

$$=\frac{5x^6+25x^4-2x^6-8x}{5(x^2-5)^{\frac{6}{5}}(x^5+4)^{\frac{4}{5}}}=\frac{3x^6+25x^4-8x}{5(x^2-5)^{\frac{6}{5}}(x^5+4)^{\frac{4}{5}}}=\frac{x(5x^4+25x^3-8)}{5(x^2-5)^{\frac{6}{5}}(x^5+4)^{\frac{4}{5}}}$$

6. $f(x)=\sqrt[4]{\dfrac{x^3+8}{x^3-8}}=\left(\dfrac{x^3+8}{x^3-8}\right)^{\frac{1}{4}}$

Solution.

$$f'(x)=\frac{\frac{1}{4}(3x^2)(x^3+8)^{-\frac{3}{4}}(x^3-8)^{\frac{1}{4}}}{(x^3-8)^{\frac{2}{4}}}-\frac{\frac{1}{4}(3x^2)(x^3-8)^{-\frac{3}{4}}(x^3+8)^{\frac{1}{4}}}{(x^3-8)^{\frac{2}{4}}}$$

$$=\frac{\frac{(3x^2)(x^3-8)^{\frac{1}{4}}}{4(x^3+8)^{\frac{3}{4}}}-\frac{(3x^2)(x^3+8)^{\frac{1}{4}}}{4(x^3-8)^{\frac{3}{4}}}}{(x^3-8)^{\frac{2}{4}}}=\frac{(3x^2)(x^3-8)-(3x^2)(x^3+8)}{4(x^3+8)^{\frac{3}{4}}(x^3-8)^{\frac{5}{4}}}$$

$$=\frac{3x^5-24x^2-3x^5-24x^2}{4(x^3+8)^{\frac{3}{4}}(x^3-8)^{\frac{5}{4}}}=\frac{-48x^2}{4(x^3+8)^{\frac{3}{4}}(x^3-8)^{\frac{5}{4}}}=-\frac{12x^2}{(x^3+8)^{\frac{3}{4}}(x^3-8)^{\frac{5}{4}}}$$

7. $f(x)=\dfrac{\sqrt{x+2}}{\sqrt{x-2}}=\dfrac{(x+2)^{\frac{1}{2}}}{(x-2)^{\frac{1}{2}}}$

Solution.

$$f'(x)=\frac{\left(\frac{1}{2}(x+2)^{-\frac{1}{2}}(1)\right)\left((x-2)^{\frac{1}{2}}(1)-(x+2)^{\frac{1}{2}}(1)\right)}{(x-2)}$$

$$=\frac{(x+2)^{-\frac{1}{2}}(x-2)^{\frac{1}{2}}}{2(x-2)}-\frac{(x+2)^{-\frac{1}{2}}(x+2)^{\frac{1}{2}}}{2(x-2)}$$

$$=\frac{(x-2)^{\frac{1}{2}}}{2(x-2)(x+2)^{\frac{1}{2}}}-\frac{(x+2)^{\frac{1}{2}}}{2(x-2)(x+2)^{\frac{1}{2}}}$$

$$=\frac{(x-2)-(x+2)}{2(x-2)^{\frac{3}{2}}(x+2)^{\frac{1}{2}}}=-\frac{4}{2(x-2)^{\frac{3}{2}}(x+2)^{\frac{1}{2}}}=-\frac{2}{(x-2)^{\frac{3}{2}}(x+2)^{\frac{1}{2}}}$$

8. $f(x)=\dfrac{(x-1)^4}{(x^2-2x)^5}$

Solution.

$$f'(x)=\left(\frac{4(x-1)^3(1)}{(x^2-2x)^5}\right)\left(\frac{(x^2-2x)^5(1)-(x-1)^4 5(x^2-2x)^4(2x-2)}{(x^2-2x)^5}\right)$$

$$=\frac{4(x-1)^3-(x-1)^4(5)(2x-2)}{(x^2-2x)^{10}}=\frac{4(x-1)^3}{(x^2-2x)^5}-\frac{5(x-1)^4(2x-2)}{(x^2-2x)^6}$$

$$=\frac{(x-1)^3\left(4(x^2-2x)-5(x-1)(2x-2)\right)}{(x^2-2x)^6}$$

$$=\frac{(x-1)^3\left(4x^2-8x-10x^2+20x-10\right)}{(x^2-2x)^6}$$

$$=\frac{(x-1)^3\left(-6x^2+12x-10\right)}{(x^2-2x)^6}=-\frac{2(x+1)^3(3x^2-6x+5)}{(x^2-2x)^6}$$