# DERIVATIVES OF LOGARITHMIC FUNCTIONS

**Note:**

1. $y=\ln x,\ y'=\dfrac{1}{x}$

2. $y=\ln\left(f\left(x\right)\right), y'=\dfrac{1}{f\left(x\right)}*f'\left(x\right)$

3. $y=\log_a u, y'=\dfrac{u'}{u\ln a}$

**Find the derivative of the following functions.**

1. $y=\log_e\left(x^2+10\right)$
2. $y=\log_e\left(\cos\theta\right)$
3. $y=\log_2\left(1-3x\right)$
4. $y=\ln\dfrac{\left(2t-1\right)^3}{\left(3t-1\right)^4}$
5. $y=\dfrac{\ln u}{1+\ln\left(2u\right)}$
6. $y=5\ln\left(x^2+7x+1\right)$
7. $y=\left(8x-3\right)\ln\left(2x^2+5\right)$
8. $y=e^{-x}\ln\left(x+2\right)$
9. $y=\left(e^{9x}+3\right)\ln\left(4x^2+11\right)$
10. $y=\dfrac{\ln\left(3x+1\right)}{x-7}$
11. $y=\left[\ln\left(2x-1\right)\right]^3$
12. $y=\ln\left(t^2+1\right)$
13. $y=\ln\left(e^{2x}\right)$
14. $y=\ln\left(\ln t\right)+\ln\left(\ln 2\right)$
15. $y=\ln\left(e^{7x}\right)$
16. $y=\ln\left(e^{\ln t}\right)$
17. $y=x\ln x-x+2$
18. $y=2x\left(\ln x+\ln 2\right)-2x+e$
19. $y=\ln\left(\sin x+\cos x\right)$
20. $y=\dfrac{e^{2x}}{\ln x}$

## SOLUTIONS

1. $y=\log_e\left(x^2+10\right)$

$$y'=\frac{2x}{\left(x^2+10\right)\ln e}=\frac{2x}{x^2+10}$$

2. $y=\log_e\left(\cos\theta\right)$

$$y'=\frac{1}{\left(\cos\theta\right)}*-\sin\theta=-\sec\left(\theta\right)\sin\left(\theta\right)$$

3. $y=\log_2(1-3x)$

$$y'=\frac{-3}{(1-3x)\ln 2}$$

4. $y=\ln\frac{(2t-1)^3}{(3t-1)^4}$

$$y'=\frac{(3t-1)^4}{(2t-1)^3}*\frac{6(2t-1)^2(3t-1)^4-12(3t-1)^3(2t-1)^3}{(3t-1)^8}$$

$$=\frac{1}{(2t-1)^3}*\frac{6(2t-1)^2(3t-1)^3\{(3t-1)-2(2t-1)\}}{(3t-1)^4}$$

$$=\frac{6(3t-1-2(2t-1))}{(2t-1)(3t-1)}=\frac{6(3t-1-4t+2)}{(2t-1)(3t-1)}=-\frac{6(t-1)}{(2t-1)(3t-1)}$$

5. $y=\frac{\ln u}{1+\ln(2u)}$

$$y'=\frac{\left(\frac{1}{u}\right)(1+\ln(2u))-\ln u\left(\frac{1}{u}\right)}{(1+\ln(2u))^2}=\frac{\frac{1+\ln(2u)}{u}-\frac{\ln u}{u}}{(1+\ln(2u))^2}$$

$$=\frac{\frac{1+\ln\left(\frac{2u}{u}\right)}{u}}{(1+\ln(2u))^2}=\frac{\frac{1+\ln 2}{u}}{(1+\ln(2u))^2}=\frac{1+\ln 2}{(1+\ln(2u))^2}$$

6. $y=5\ln\left(x^2+7x+1\right)$

$$y'=5*\frac{1}{x^2+7x+1}*2x+7$$

$$=\frac{5(2x+7)}{x^2+7x+1}$$

7. $y=(8x-3)\ln\left(2x^2+5\right)$

$$y'=(8x-3)\ln\left(\frac{1}{2x^2+5}\right)4x+8\ln\left(2x^2+5\right)$$

$$=\frac{4x(8x-3)}{2x^2+5}+8\ln\left(2x^2+5\right)$$

8. $y = e^{-x}\ln(x+2)$

$$y' = \frac{e^{-x}}{x+2} + \left(-e^{-x}\right)\left(\ln(x+2)\right)$$

$$= \frac{e^{-x}}{x+2} - \frac{e^{-x}\left(\ln(x+2)\right)(x+2)}{x+2}$$

$$= \frac{e^{-x} - e^{-x}\ln(x+2)x - e^{-x}\ln(x+2)2}{x+2}$$

$$= \frac{e^{-x} - e^{-x}\ln(x+2) - e^{-x}2\ln(x+2)}{x+2}$$

$$= \frac{e^{-x} - e^{-x}\ln(x+2)x - e^{-x}\ln(x+2)^2}{x+2}$$

9. $y = \left(e^{9x}+3\right)\ln\left(4x^2+11\right)$

$$y' = \left(e^{9x}+3\right)\left(\frac{1}{4x^2+11}\right)8x + 9e^{9x}\ln\left(4x^2+11\right)$$

$$= \frac{8x\left(e^{9x}+3\right)}{4x^2+11} + 9e^{9x}\ln\left(4x^2+11\right)$$

10. $y = \dfrac{\ln(3x+1)}{x-7}$

$$y' = \frac{(x-7)\left(\frac{1}{3x+1}\right)*3 - \ln(3x+1)}{(x-7)^2}$$

$$= \frac{\frac{3(x-7)}{3x+1} - \ln(3x+1)}{(x-7)^2} = \frac{\frac{3(x-7)}{3x+1} - \frac{\{\ln(3x+1)\}(3x+1)}{3x+1}}{(x-7)^2}$$

$$= \frac{\frac{3x-21-3x\ln(3x+1)-\ln(3x+1)}{3x+1}}{(x-7)^2}$$

$$= \frac{3x-21-x\ln(x+1)^3-\ln(3x+1)}{(3x+1)(x-7)^2}$$

11. $y = \left[\ln(2x-1)\right]^3$

$$y' = 3\ln^2(2x-1)*\frac{1}{2x-1}*2$$

$$= \frac{6\ln^2(2x-1)}{2x-1}$$

12. $y=\ln\left(t^2+1\right)$

$$y'=\frac{1}{t^2+1}*2t=\frac{2t}{t^2+1}$$

13. $y=\ln\left(e^{2x}\right)$

$$=2x\ln e=2x$$

$$y'=2$$

14. $y=\ln\left(\ln t\right)+\ln\left(\ln 2\right)$

$$y'=\frac{1}{\ln t}*\frac{1}{t}+\frac{1}{\ln 2}*0=\frac{1}{t\ln t}$$

15. $y=\ln\left(e^{7x}\right)=7x\ln e=7x$

$$y'=7$$

16. $y=\ln\left(e^{\ln t}\right)=\ln t\ln e=\ln t$

$$y'=\frac{1}{t}$$

17. $y=x\ln x-x+2$

$$y'=\ln x+x*\frac{1}{x}-1=\ln x+1-1=\ln x$$

18. $y=2x\left(\ln x+\ln 2\right)-2x+e\ =2x\ln\left(2x\right)-2x+e$

$$y'=2\left(x*\frac{1}{2x}*2+\ln\left(2x\right)\right)-2x+0$$

$$=2\left(\frac{x}{2x}*2+\ln\left(2x\right)\right)-2=2\left(1+\ln\left(2x\right)\right)-2$$

$$=2+2\ln\left(2x\right)-2=2\ln\left(2x\right)$$

19. $y=\ln\left(\sin x+\cos x\right)$

$$y'=\frac{1}{\sin x+\cos x}*\cos x-\sin x$$

$$=\frac{\cos x-\sin x}{\sin x+\cos x}$$

20. $y=\dfrac{e^{2x}}{\ln x}$

$$y'=\frac{\ln x*e^{2x}*2-e^{2x}*\frac{1}{x}}{\left(\ln x\right)^2}=\frac{2\ln x*e^{2x}-\frac{e^{2x}}{x}}{\left(\ln x\right)^2}=\frac{\ln\left(x^2\right)*e^{2x}-\frac{e^{2x}}{x}}{\left(\ln x\right)^2}$$