

CALCULUS 2

WORKED EXAMPLES

DERIVATIVES OF EXPONENTIAL FUNCTIONS

DERIVATIVES OF EXPONENTIAL FUNCTIONS

Note: 1. $\frac{d}{dx}(e^x) = e^x$. 2. $\frac{d}{dx}(a^x) = (\ln a)a^x$

Find the derivatives of the following exponential functions.

1) $f(x) = 2e^x + x^2$

2) $y = 5t^2 + 4e^t$

3) $y = 5^x + 2$

4) $f(x) = 2^x + 2 \cdot 3^x$

5) $y = 5x^2 + 2^x + 3$

6) $f(x) = 12e^x + 11^x$

7) $y = 4 \cdot 10^x - x^3$

8) $y = 3x - 2 \cdot 4^x$

9) $y = \frac{3^x}{3} + \frac{33}{\sqrt{x}}$

10) $f(x) = e^2 + x^e$

11) $f(x) = e^{1+x}$

12) $f(t) = e^{t+2}$

13) $f(t) = e^{\theta-1}$

14) $z = (\ln 4)e^x$

15) $z = (\ln 4)4^x$

16) $f(t) = (\ln 3)^t$

17) $f(x) = x^3 + 3^x$

18) $y = 5.5^t + 6.6^t$

19) $y = \pi^2 + \pi^x$

20) $h(z) = (\ln 2)^z$

21) $f(x) = e^\pi + \pi^x$

22) $f(x) = \pi^x + x^\pi$

23) $y = a^x + x^a$

24) $f(x) = x^{\pi^2} + (\pi^2)^x$

25) $f(z) = (\ln 3)z^2 + (\ln 4)e^z$

26) $g(x) = 2x - \frac{1}{\sqrt[3]{x}} + 3^x - e$

27) $g(x) = x^k + k^x$

28) $r(\theta) = e^{(e^\theta + e^{-\theta})}$

29) $g(x) = e^{3x^2} (3x^5 - 4)$

30) $f(x) = \frac{x^4 + 3}{e^{5x^5}}$

SOLUTIONS

$$1) f(x) = 2e^x + x^2$$
$$f'(x) = 2e^x + 2x$$

$$2) y = 5t^2 + 4e^t$$
$$y' = 10t + 4e^t$$

$$3) y = 5^x + 2$$
$$y' = (\ln 5)5^x$$

$$4) f(x) = 2^x + 2 \cdot 3^x$$
$$f'(x) = (\ln 2)2^x + 2(\ln 3)3^x$$

$$5) y = 5x^2 + 2^x + 3$$
$$y' = 10x + (\ln 2)2^x$$

$$6) f(x) = 12e^x + 11^x$$
$$f'(x) = 12e^x + (\ln 11)11^x$$

$$7) y = 4 \cdot 10^x - x^3$$
$$y' = 4(\ln 10)10^x - 3x^2$$

$$8) y = 3x - 2 \cdot 4^x$$
$$y' = 3 - 2(\ln 4)4^x$$

$$9) y = \frac{3^x}{3} + \frac{33}{\sqrt{x}}$$
$$y = \frac{1}{3} \cdot 3^x + 33 \cdot x^{-\frac{1}{2}}$$
$$y' = \frac{1}{3}(\ln 3)3^x - \frac{1}{2}x^{-\frac{3}{2}}(33)$$

$$10) f(x) = e^2 + x^e$$
$$f'(x) = ex^{e-1}$$

$$11) f(x) = e^{1+x}$$

$$f(x) = e^1 \cdot e^x$$
$$f'(x) = e^{1+x}$$

$$12) f(t) = e^{t+2}$$
$$f(t) = e^t \cdot e^2$$
$$f'(t) = e^{t+2}$$

$$13) f(t) = e^{\theta-1}$$
$$y = e^\theta \cdot e^{-1}$$
$$y' = e^{\theta-1}$$

$$14) z = (\ln 4)e^x$$
$$z' = (\ln 4)e^x$$

$$15) z = (\ln 4)4^x$$
$$z' = (\ln 4)(\ln 4)4^x$$
$$z' = (\ln 4)^2 4^x$$

$$16) f(t) = (\ln 3)^t$$
$$f'(t) = (\ln 3)^t \ln(\ln 3)$$

$$17) f(x) = x^3 + 3^x$$
$$f'(x) = 3x^2 + (\ln 3)3^x$$

$$18) y = 5.5^t + 6.6^t$$
$$y' = 5.5^t (\ln 5.5) + 6.6^t (\ln 6.6)$$

$$19) y = \pi^2 + \pi^x$$
$$y' = \pi^x \ln \pi$$

$$20) h(z) = (\ln 2)^z$$
$$h'(z) = (\ln 2)^z \ln(\ln 2)$$

$$21) f(x) = e^\pi + \pi^x$$

$$f'(x) = \pi^x \ln \pi$$

$$22) f(x) = \pi^x + x^\pi$$

$$f'(x) = \pi^x \ln \pi + \pi x^{\pi-1}$$

$$23) y = a^x + x^a$$

$$y' = a^x \ln a + ax^{a-1}$$

$$24) f(x) = x^{\pi^2} + (\pi^2)^x$$

$$f'(x) = \pi^2 x^{\pi^2-1} + (\pi^2)^x \ln(\pi^2)$$

$$25) f(z) = (\ln 3)z^2 + (\ln 4)e^z$$

$$f'(z) = (2 \ln 3)z + (\ln 4)e^z$$

$$26) g(x) = 2x - \frac{1}{\sqrt[3]{x}} + 3^x - e$$

$$g'(x) = 2 + \frac{x^{-\frac{4}{3}}}{3} + 3^x \ln 3$$

$$27) g(x) = x^k + k^x$$

$$g'(x) = kx^{k-1} + k^x \ln k$$

$$28) r(\theta) = e^{(e^\theta + e^{-\theta})}$$

$$r'(\theta) = (e^\theta - e^{-\theta}) \left(e^{(e^\theta + e^{-\theta})} \right)$$

$$29) g(x) = e^{3x^2} (3x^5 - 4)$$

$$\begin{aligned} g'(x) &= 6xe^{3x^2} (3x^5 - 4) + e^{3x^2} * 15x^4 \\ &= 3xe^{3x^2} (2(3x^5 - 4) + 5x^3) \\ &= 3xe^{3x^2} (6x^5 + 5x^3 - 8) \end{aligned}$$

$$30) f(x) = \frac{x^4 + 3}{e^{5x^5}}$$

$$\begin{aligned} f'(x) &= \frac{e^{5x^5} * 4x^3 - (x^4 + 3) * 25x^4 e^{5x^5}}{(e^{5x^5})^2} \\ &= \frac{x^3 e^{5x^5} \{4 - 25x(x^4 + 3)\}}{(e^{5x})^2} \\ &= \frac{x^3 (4 - 25x^5 - 75x)}{e^{5x^5}} \end{aligned}$$