

LESSON 1

SOLVING BY ELIMINATION

When solving systems of equation with three variables we use one of two regular methods: the elimination method or the substitution method. We will first look at the elimination method.

Steps to solve a linear system of three variables by elimination:

1. Pick any pair of equations and solve for one variable.
2. Pick another pair of equations and solve for the same variable.
3. A system of two equations in two unknowns is now created. Solve the resulting two equations.
4. Back-substitute known variables into any one of the original equations and solve for the missing variables.

Example 1

$$5x - 2y - 4z = 3 \text{ equation 1}$$

$$3x + 3y + 2z = -3 \text{ equation 2}$$

$$-2x + 5y + 3z = 3 \text{ equation 3}$$

Step 1: Pick any pair of equations and solve for one variable.

We choose to eliminate y using equation 1 and equation 2. Proceed by multiplying equation 1 by 3 and equation 2 by 2.

$$5x - 2y - 4z = 3 \text{ equation 1} \times 3 \Rightarrow 15x - 6y - 12z = 9$$

$$3x + 3y + 2z = -3 \text{ equation 2} \times 2 \Rightarrow 6x + 6y + 4z = -6$$

By addition.

$$\begin{array}{r} 15x - 6y - 12z = 9 \\ 6x + 6y + 4z = -6 \\ \hline 21x - 8z = 3 \text{ equation 4} \end{array}$$

Step 2: Pick another pair of equations and solve for the same variable.

Here we choose equation 2 and equation 3. Multiply equation 2 by -5 and equation 3 by 3.

$$3x + 3y + 2z = -3 \text{ equation 2} \times -5 \Rightarrow -15x - 15y - 10z = 15$$

$$-2x + 5y + 3z = 3 \text{ equation 3} \times 3 \Rightarrow -6x + 15y + 9z = 9$$

By addition.

$$\begin{array}{r} -15x - 15y - 10z = 15 \\ -6x + 15y + 9z = 9 \\ \hline -21x - z = 24 \text{ equation 5} \end{array}$$

Step 3: A system of two equations in two unknowns is now created. Solve the resulting two equations.

$$21x - 8z = 3 \text{ equation 4}$$

$$-21x - z = 24 \text{ equation 5}$$

$$-9z = 27 \Rightarrow z = -3$$

Step 4: Back-substitute known variables into any one of the original equations and solve for the missing variables.

Substitute $z = -3$ in equation 1, $5x - 2y - 4z = 3$

$$5x - 2y - 4z = 3$$

$$5x - 2y - 4(-3) = 3$$

$$5x - 2y = -9$$

Next, substitute $z = -3$ in equation 2.

$$3x + 3y + 2z = -3$$

$$3x + 3y + 2(-3) = -3$$

$$3x + 3y = 3$$

$$5x - 2y = -9 \times 3 \Rightarrow 15x - 6y = -27$$

$$3x + 3y = 3 \times 2 \Rightarrow 6x + 6y = 6$$

$$\begin{array}{r} \text{-----} \\ 21x = -21 \Rightarrow x = -1 \end{array}$$

Next, substitute $x = -1$, and $z = -3$ in equation 3 to solve for y .

$$-2x + 5y + 3z = 3$$

$$-2(-1) + 5y + 3(-3) = 3$$

$$2 + 5y - 9 = 3$$

$$5y = 10 \Rightarrow y = 2$$

Thus, $x = -1$, $y = 2$, and $z = -3$

Substitute the three values in any one of the equations. We select equation 3.

$$-2x + 5y + 3z = 3 \Rightarrow -2(-1) + 5(2) + 3(-3) = 3 \Rightarrow 2 + 10 - 9 = 3.$$

Example 2

$$-3a - b - 3c = -8 \text{ equation 1}$$

$$-5a + 3b + 6c = -4 \text{ equation 2}$$

$$-6a - 4b + c = -20 \text{ equation 3}$$

Step 1: Pick any pair of equations and solve for one variable.

We choose to eliminate b using equation 1 and equation 2. Proceed by multiplying equation 1 by 3 and leaving equation 2 as it is.

$$-3a - b - 3c = -8 \text{ equation 1} \times 3 \Rightarrow -9a - 3b - 9c = -24$$

$$-5a + 3b + 6c = -4 \text{ equation 2} \Rightarrow -5a + 3b + 6c = -4$$

$$\begin{array}{r} \text{-----} \\ -14a - 3c = -28 \text{ equation 4 By addition} \end{array}$$

Step 2: Pick another pair of equations and solve for the same variable.

Here we choose equation 2 and equation 3. Multiply equation 2 by 4 and equation 3 by 3.

$$-5a + 3b + 6c = -4 \text{ equation 2 } \times 4 \Rightarrow -20a + 12b + 24c = -16$$

$$-6a - 4b + c = -20 \text{ equation 3 } \times 3 \Rightarrow -18a - 12b + 3c = -60$$

$$\begin{array}{r} \text{-----} \\ -38a + 27c = -76 \text{ equation 5 By addition} \end{array}$$

Step 3: A system of two equations in two unknowns is now created. Solve the resulting two equations.

$$-14a - 3c = -28 \text{ equation 4 } \times 9 \Rightarrow -126a - 27c = -252$$

$$-38a + 27c = -76 \text{ equation 5 } \Rightarrow -38a + 27c = -76$$

$$\begin{array}{r} \text{-----} \\ -164a = -328 \text{ By addition} \end{array}$$

$$a = 2$$

Step 4: Back-substitute known variables into any one of the original equations and solve for the missing variables.

Now substitute $a = 2$ in equation 1, $-3a - b - 3c = -8$.

$$-3(2) - b - 3c = -8$$

$$-6 - b - 3c = -8$$

$$-b - 3c = -2 \text{ equation 6}$$

Next substitute $a = 2$ in equation 2, $-5a + 3b + 6c = -4$.

$$-5a + 3b + 6c = -4$$

$$-5(2) + 3b + 6c = -4$$

$$-10 + 3b + 6c = -4$$

$$3b + 6c = 6 \text{ equation 7}$$

Solving equations 6 and 7 we obtain, $b = 2$.

To solve for c , we substitute the values for a and b in any of the three original equations. We use equation 3 for this purpose.

$$-5a + 3b + 6c = -4$$

$$-5(2) + 3(2) + 6c = -4$$

$$-10 + 3(2) + 6c = -4$$

$$3b + 6c = 6 \text{ equation 7}$$

Thus, $a = 2$, $b = 2$, and $c = 0$.

Substitute the three values in any one of the equations. We select equation 2.

$$-5a + 3b + 6c = -4 \Rightarrow -5(2) + 3(2) + 6(0) = -4 \Rightarrow -10 + 6 + 0 = -4.$$

Lesson 1 Exercise

Solve each system by elimination.

$$\begin{aligned} 1) \quad & -x - 5y - 5z = 2 \\ & 4x - 5y + 4z = 19 \\ & x + 5y - z = -20 \end{aligned}$$

$$\begin{aligned} 2) \quad & -4x - 5y - z = 18 \\ & -2x - 5y - 2z = 12 \\ & -2x + 5y + 2z = 4 \end{aligned}$$

$$\begin{aligned} 3) \quad & -x - 5y + z = 17 \\ & -5x - 5y + 5z = 5 \\ & 2x + 5y - 3z = -10 \end{aligned}$$

$$\begin{aligned} 4) \quad & 4x + 4y + z = 24 \\ & 2x - 4y + z = 0 \\ & 5x - 4y - 5z = 12 \end{aligned}$$

$$\begin{aligned} 5) \quad & 4r - 4s + 4t = -4 \\ & 4r + s - 2t = 5 \\ & -3r - 3s - 4t = -16 \end{aligned}$$

$$\begin{aligned} 6) \quad & x - 6y + 4z = -12 \\ & x + y - 4z = 12 \\ & 2x + 2y + 5z = -15 \end{aligned}$$

$$\begin{aligned} 7) \quad & x - y - 2z = -6 \\ & 3x + 2y = -25 \\ & -4x + y - z = 12 \end{aligned}$$

$$\begin{aligned} 8) \quad & 5a + 5b + 5c = -20 \\ & 4a + 3b + 3c = -6 \\ & -4a + 3b + 3c = 9 \end{aligned}$$