LESSON 3

DIVIDING COMPLEX NUMBERS

To divide complex numbers, you must multiply by the conjugate. To find the conjugate of a complex number all you have to do is change the sign between the two terms in the denominator. Next distribute in both the numerator and denominator to remove the parenthesis.

Example 1.

$$\frac{-3-2i}{-10-3i}$$

1) **Find the conjugate.** The conjugate of **$-10-3i$ is $-10+3i$.**
2) **Distribute in both numerator and denominator.**
3) **Simplify.**

$$\frac{-3-2i}{-10-3i}\times\frac{-10+3i}{-10+3i}\Rightarrow\frac{30-9i+20i-6i^2}{100-30i+30i-9i^2}\Rightarrow\frac{36+11i}{109}$$

Example 2.

$$\frac{3-9i}{5-8i}$$

$$\frac{-3-9i}{5-8i}\times\frac{5+8i}{5+8i}\Rightarrow\frac{-15-24i-45i-72i^2}{25+40i-40i-64i^2}\Rightarrow\frac{57}{89}-\frac{69i}{89}$$

Example 3.

$$\frac{4+i}{8+9i}$$

$$\frac{4+i}{8+9i}\times\frac{8-9i}{8-9i}\Rightarrow\frac{32-36i+8i-9i^2}{64-72i+72i-81i^2}\Rightarrow\frac{41}{145}-\frac{28i}{145}$$

Example 4.

$$\frac{3+9i}{-6-6i}$$

$$\frac{3+9i}{-6-6i}\Rightarrow\frac{3(1+3i)}{3(-2-2i)}\Rightarrow\frac{1+3i}{-2-2i}\times\frac{-2+2i}{-2+2i}$$

$$\frac{-2+2i-6i+6i^2}{4-4i+4i-4i^2}\Rightarrow\frac{-8-4i}{8}\Rightarrow\frac{4(-2-i)}{4(2)}\Rightarrow\frac{-2-i}{2}$$

Example 5.

$$\frac{3+2i}{4-3i}$$

$$\frac{3+2i}{4-3i}\times\frac{4+3i}{4+3i}\Rightarrow\frac{12+9i+8i+6i^2}{16+12i-12i-9i^2}\Rightarrow\frac{6+17i}{25}$$

Example 6.

$$\frac{2+i}{i}$$

$$\frac{2+i}{i}\times\frac{i}{i}\Rightarrow\frac{2i+i^2}{i^2}\Rightarrow\frac{-1+2i}{-1}\Rightarrow 1-2i$$

Example 7.

$$\frac{2+4i}{i}$$

$$\frac{2+4i}{i}\times\frac{i}{i}\Rightarrow\frac{2i+4i^2}{i^2}\Rightarrow\frac{2i-4}{-1}\Rightarrow 4-2i$$

Lesson 3 Exercise

Dividing Complex Numbers

Simplify.

1) $\frac{5}{-5i}$

2) $\frac{1}{-2i}$

3) $-\frac{2}{i}$

4) $\frac{7}{4i}$

5) $\frac{4+i}{8i}$

6) $\frac{-5-i}{-10i}$

7) $\frac{9+i}{-7i}$

8) $\frac{6-6i}{-4i}$

9) $\frac{2i}{3-9i}$

10) $\frac{i}{2-3i}$

11) $\frac{5i}{6+8i}$

12) $\frac{10}{10+5i}$

13) $\frac{-1+5i}{-8-7i}$

14) $\frac{-2-9i}{-2+7i}$

15) $\frac{4+i}{2-5i}$

16) $\frac{5-6i}{-5+10i}$

SOLUTIONS

Lesson 1 Exercise

1. $7 - 2i$. **2**. $7i$. **3**. 4. **4**. $3 + 3i$ **5**. $8 + i$.

6. $1 - 2i$. **7.** 5 – 2i. **8**. $2 + 4i$. **9**. $13 + 4i$. **10**. – 1.

Lesson 2 Exercise

1. $64i$. **2**. $14i$. **3**. $-18 - 6i$ **4**. $8i$. **5**. 24.

6. – 64. **7**. $-20 - 46i$. **8**. – 25 + 49i. **9**. $20 - 50i$. **10**.$18 + 66i$.

11. $2 - 18i$. **12**. $30 + 20i$. **13**. $-21 + 18i$. **14**. $-24 - 36i$. **15**. $126 + 210i$.

16. $7 - 35i$. **17.** $7 - 199i$. **18.** $568 + 144i$. **19**. $252 + 84i$. **20.** 224 + 288i.

Lesson 3 Exercise

1. i. **2**. $\frac{1}{-2i}$. **3**. $\frac{-2}{i}$. **4**. $\frac{7}{4i}$. **5**. $\frac{4+i}{8i}$.

6. $\frac{-5-i}{-10i}$. **7**. $\frac{9+i}{-7i}$. **8**. $\frac{6-6i}{-4i}$. **9**. $\frac{2i}{3-9i}$. **10.** $\frac{i}{2-3i}$.

11. $\frac{5i}{6-8i}$. **12**. $\frac{10}{10+5i}$. **13**. $\frac{-1+5i}{-8-7i}$. **14**. $\frac{-2-9i}{-2+7i}$. **15**. $\frac{4+i}{2-5i}$.

16. $\frac{5-6i}{-5+10i}$. **17.** $\frac{-3-9i}{-5-8i}$. **18**. $\frac{4+i}{8+9i}$. **19**. $\frac{-3-2i}{-10-3i}$. **20**. $\frac{3+9i}{-6-6i}$.