LESSON 2

SOLVING SYSTEMS OF LINEAR EQUATIONS: THREE VARIABLES

SOLVING BY SUBSTITUTION

Solving Systems of Equations by Substitution Method

The Substitution Method is the most elementary of all the methods of solving systems of equations. The Substitution method, as the method indicates, involves substituting something into the equations to make them much simpler to solve. So, what do we substitute? We express one of the variables in terms of the other until we have only one equation with only one variable. We then solve for that variable and after we obtain its value, we perform what is called **Back Substitution** to solve for the other missing variable(s).

Steps:

1. Select an equation and solve for one of its variables. There is no incorrect choice so you may choose any variable.
2. Substitute the value for the first variable you solved into another equation and solve for the next variable.
3. Substitute the values for the two variables that you solved into the remaining equation to solve for third variable.
4. After solving for the final variable, substitute all three values into any one of the three equation to verify whether they are solutions to the system.

Example 1

$x + 3y = -17$ equation 1

$3x = -6$ equation 2

$4x - 3y + 6z = 25$ equation 3

Step 1: Select an equation and solve for one of its variables. There is no incorrect choice so you may choose any variable.

We choose equation 2, **$3x = -6$. Then $x = -2$.**

Step 2: Substitute the value for the first variable you solved into another equation and solve for the next variable.

Substitute $x = -2$ in equation, $x + 3y = -17$.

Then, $-2 + 3y = -17$

$3y = -15$

$y = -5$

Step 3: Substitute the values for the two variables that you solved into the remaining equation to solve for third variable.

Substitute $x = -2$ and $y = -5$ in equation 3, $4x - 3y + 6z = 25$.

Then, $4(-2) - 3(-5) + 6z = 25$

$-8 + 15 + 6z = 25$

$6z = 18$

$z = 3$

Step 4: After solving for the final variable, substitute all three values into any one of the three equation to verify whether they are solutions to the system.
Substitute all three values in equation 3,

$4(-2) - 3(-5) + 6(3) = 25$

$-8 + 15 + 18 = 25$

$25 = 25$

Therefore, $x = -2$, $y = -5$, and $z = 3$.

Example 2

$-2x + y + 6z = 1$ equation 1
$3x + 2y + 5z = 16$ equation 2
$7x + 3y - 4z = 11$ equation 3

Step 1: Select an equation and solve for one of its variables. There is no incorrect choice so you may choose any variable.
We choose equation 1, $-2x + y + 6z = 1$. Then, $y = 2x - 6z + 1$.
Step 2: Substitute the value for the first variable you solved into another equation and solve for the next variable.
Substitute $y = 2x - 6z + 1$ in equation 2.
Then, $3x + 2y + 5z = 16$ becomes $3x + 2(2x - 6z + 1) + 5z = 16$

$$3x + 4x - 12z + 2 + 5z = 16$$
$$7x - 7z = 14 \Rightarrow x = z + 2$$

Step 3: Substitute the values for the two variables that you solved into the remaining equation to solve for third variable.
Substitute the values for x and y in equation 3.

Therefore $7x + 3y - 4z = 11$ becomes
$7(z + 2) + 3\{2(z + 2) - 6z + 1\} - 4z = 11$
$7z + 14 + 6z + 12 - 18z + 3 - 4z = 11$
$-9z = -18 \Rightarrow z = 2$
$x = z + 2 \Rightarrow x = 2 + 2 = 4$
$y = 2x - 6z + 1 \Rightarrow y = 2 \times 4 - 6 \times 2 + 1 \Rightarrow y = -3$.
Step 4: After solving for the final variable, substitute all three values into any one of the three equation to verify whether they are solutions to the system.
Substitute all three values in equation 2,
$3x + 2y + 5z = 16 \Rightarrow 3(4) + 2(-3) + 5(2) = 16$

$$12 - 6 + 10 = 16 \Rightarrow 16 = 16$$

Therefore $x = 4$, $y = -3$, and $z = 2$.

Lesson 2 Exercise

Solve each system by substitution.

1) $-x - y - 3z = -9$
$z = -3x - 1$
$x = 5y - z + 23$

2) $x = -4z - 19$
$y = 5x + z - 4$
$-5y - z = 25$

3) $y = x + z + 5$
$z = -3y - 3$
$2x - y = -4$

4) $-2y + 5z = -3$
$y = -5x - 4z - 5$
$x = 4z + 4$

5) $y = x + 4z - 5$
$4x + 3y - 2z = 5$
$z = -2x + 2$

6) $x = 3y - 3z + 8$
$z = 4x + 5y - 14$
$3y + 2z = 14$

7) $-5x - 3y + z = -4$
$-2x - 2y + 2z = 4$
$z = x + 5$

8) $-4x + 2z = 14$
$y = x + z + 12$
$-2x - 4z = 22$

SOLUTIONS

Lesson 1 Exercise

1. **(- 2, 3, - 3)**
2. **(- 4, 0, - 2)**
3. **(- 1, - 4, - 4)**
4. **(4, 2, 0)**
5. **(1, 3, 1)**
6. **(0, 0, - 3)**
7. **(- 5, - 5, 3)**
8. **No unique solutions**

Lesson 2 Exercise

1. **(- 2, - 4, 5)**
2. **(1, - 4, - 5)**
3. **(- 2, 0, - 3)**
4. **(0, - 1, - 1)**
5. **(0, 3, 2)**
6. **(2, 2, 4)**
7. **(0, 3, 5)**
8. **(- 5, 4, - 3)**