LESSON 1

SOLVING SYSTEMS OF LINEAR EQUATIONS: THREE VARIABLES

SOLVING BY ELIMINATION

When solving systems of equation with three variables we use the elimination method or the substitution method to make a system of two equations in two variables.

Steps to solve a linear system of three variables by elimination:

1. Pick any pair of equations and solve for one variable.
2. Pick another pair of equations and solve for the same variable.
3. A system of two equations in two unknowns is now created. Solve the resulting two equations.
4. Back-substitute known variables into any one of the original equations and solve for the missing variables.

Example 1

$5x-2y-4z=3$ equation 1

$3x+3y+2z=-3$ equation 2

$-2x+5y+3z=3$ equation 3

Step 1: Pick any pair of equations and solve for one variable.
We choose to eliminate y using equation 1 and equation 2. Proceed by multiplying equation 1 by 3 and equation 2 by 2.

$5x-2y-4z=3$ equation 1 x 3 $\Rightarrow 15x-6y-12z=9$

$3x+3y+2z=-3$ equation 2 x 2 $\Rightarrow 6x+6y+4z=-6$

$21x-8z=3$ equation 4

Step 2: Pick another pair of equations and solve for the same variable.

Here we choose equation 2 and equation 3. Multiply equation 2 by 5 and equation 3 by 3.

$3x+3y+2x=-3$ equation 2 x -5 $\Rightarrow -15x-15y-10z=15$

$-2x+5y+3z=3$ equation 3 x 3 $\Rightarrow -6x+15y+9z=9$

$-21x-z=24$ equation 5

Step 3: A system of two equations in two unknowns is now created. Solve the resulting two equations.

$21x - 8z = 3$ equation 4

$-21x - z = 24$ equation 5

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$-9z = 27 \Rightarrow z = -3$

Step 4: Back-substitute the known variables into any one of the original equations and solve for the missing variables.

Substitute ***z* = - 3** in equation 1, **5*x* – 2*y* – 4*z* = 3**

$5x - 2y - 4z = 3$

$5x - 2y - 4(-3) = 3$

$5x - 2y = -9$

Next, substitute *z* = - 3 in equation 2.

$3x + 3y + 2z = -3$

$3x + 3y + 2(-3) = -3$

$3x + 3y = 3$

$5x - 2y = -9$ x 3 $\Rightarrow 15x - 6y = -27$

$3x + 3y = 3$ x 2 $\Rightarrow 6x + 6y = 6$

$21x = -21 \Rightarrow x = -1$

Next, substitute x = - 1, and *z* = - 3 in equation 3 to solve for y.

$-2x + 5y + 3z = 3$

$-2(-1) + 5y + 3(-3) = 3$

$2 + 5y - 9 = 3$

$5y = 10 \Rightarrow y = 2$

Thus, *x* = - 1, *y* = 2, and *z* = - 3

Example 2

- 3a – b – 3c = - 8

- 5a + 3b + 6c = -4

- 6a – 4b + c = - 20

Step 1: Pick any pair of equations and solve for one variable.

We choose to eliminate *b* using equation 1 and equation 2. Proceed by multiplying equation 1 by 3 and leaving equation 2 as it is.

$-3a - b - 3c = -8$ equation 1 x 3 $\Rightarrow -9a - 3b - 9c = -24$

$-5a + 3b + 6c = -4$ equation 2 $\Rightarrow -5a + 3b + 6c = -4$

$-14a - 3c = -28$ equation 4 By addition

Step 2: Pick another pair of equations and solve for the same variable.

Here we choose equation 2 and equation 3. Multiply equation 2 by 4 and equation 3 by 3.

$-5a+3b+6c=-4$ equation 2 x 4 $\Rightarrow -20a+12b+24c=-16$

$-6a-4b+c=-20$ equation 3 x 3 $\Rightarrow -18a-12b+3c=-60$

$-38a+27c=-76$ equation 5 By addition

Step 3: A system of two equations in two unknowns is now created. Solve the resulting two equations.

$-14a-3c=-28$ equation 4 x 9 $\Rightarrow -126a-27c=-252$

$-38a+27c=-76$ equation 5 $\Rightarrow -38a+27c=-76$

$-164a=-328$ By addition

$a=2$

Step 4: Back-substitute known variables into any one of the original equations and solve for the missing variables.

Now substitute *a* = 2 in equation 1, - 3*a* – *b* – 3*c* = - 8.

$-3(2)-b-3c=-8$

$-6-b-3c=-8$

$-b-3c=-2$ equation 6

Next substitute *a* = 2 in equation 2, -5*a* + 3*b* + 6*c* = - 4.

$-5a+3b+6c=-4$

$-5(2)+3b+6c=-4$

$-10+3b+6c=-4$

$3b+6c=6$ equation 7

Solving equations 6 and 7 we obtain, *b* = 2.
To solve for ***c*,** we substitute the values for ***a*** and ***b*** in any of the three original equations. We use equation 3 for this purpose.

$-5a+3b+6c=-4$

$-5(2)+3b+6c=-4$

$-10+3b+6c=-4$

$3b+6c=6$ equation 7

Thus, $a = 2$, $b = 2$, and $c = 0$.

Lesson 1 Exercise

Solve each system by elimination.

1) $-x - 5y - 5z = 2$
$4x - 5y + 4z = 19$
$x + 5y - z = -20$

2) $-4x - 5y - z = 18$
$-2x - 5y - 2z = 12$
$-2x + 5y + 2z = 4$

3) $-x - 5y + z = 17$
$-5x - 5y + 5z = 5$
$2x + 5y - 3z = -10$

4) $4x + 4y + z = 24$
$2x - 4y + z = 0$
$5x - 4y - 5z = 12$

5) $4r - 4s + 4t = -4$
$4r + s - 2t = 5$
$-3r - 3s - 4t = -16$

6) $x - 6y + 4z = -12$
$x + y - 4z = 12$
$2x + 2y + 5z = -15$

7) $x - y - 2z = -6$
$3x + 2y = -25$
$-4x + y - z = 12$

8) $5a + 5b + 5c = -20$
$4a + 3b + 3c = -6$
$-4a + 3b + 3c = 9$