

# LESSON 1

## **SOLVING SYSTEMS OF LINEAR EQUATIONS: TWO UNKNOWNNS**

A system of linear equations is two or more linear equations that are being solved simultaneously. In this lesson we will be looking at systems that have only two linear equations and two unknown variables.

There are three ways to solve systems of linear equations: substitution, elimination, and graphing. Also, there are three possible outcomes that may follow from the solution of systems of linear equations: no solution, one solution, and infinite solutions.

NOTE:

**If a system has at least one solution, it is said to be *consistent* .**

**If a *consistent* system has exactly one solution, it is *independent* .**

### Solving systems of linear equations by substitution.

#### **Example 1.**

Solve by substitution

$$y = 6x - 11$$

$$-2x - 3y = -7$$

**Step 1:** Substitute  $6x - 11$  for  $y$  in the second equation since  $y = 6x - 11$ . Then we will have,

$$-2x - 3(6x - 11) = -7$$

**Step 2: Solve**

$$-2x - 18x + 33 = -7$$

$$-20x + 33 = -7$$

$$-20x = -40$$

$x = 2$  To find the value of  $y$  substitute  $2$  in the equation  $y = 6x - 11$ , then we have

$$y = 6(2) - 11 = 1$$

#### **Example 2.**

Solve by substitution

$$2x - 3y = -1$$

$$y = x - 1$$

**Step 1:** Substitute  $x - 1$  for  $y$  in the first equation since  $y = x - 1$ . Then we will have,

$$2x - 3(x - 1) = -1$$

**Step 2: Solve**

$$2x - 3x + 3 = -1$$

$$-x + 3 = -1$$

$x = 4$  To find the value of  $y$  substitute  $4$  in the equation  $y = x - 1$ , then we have

$$y = 4 - 1 = 3$$

#### **Example 3.**

Solve by substitution

$$2x + y = 20$$

$$6x - 5y = 12$$

This is slightly different because it is not clear what to substitute where. However, from inspection it appears easier to substitute the first equation into the second.

**Step 1:** Re-arrange the equation in  $y = mx + b$  form

$$\begin{array}{r} 2x + y = 20 \\ -2x \quad -2x \\ \hline \end{array}$$

$$y = -2x + 20$$

**Step 2:** Substitute  $-2x + 20$  for  $y$  in the second equation since  $y = -2x + 20$ . Then we will have,

$$6x - 5(-2x + 20) = 12$$

**Step 3: Solve**

$$6x + 10x - 100 = 12$$

$$16x - 100 = 12$$

$$16x = 112$$

$$x = 7, \text{ then } y = 6$$

### Solving systems of linear equations by elimination.

#### **Example 1.**

Solve by elimination

$$3x + y = -21$$

$$x + y = -5$$

**Step 1:** Eliminate  $y$  using subtraction.

$$3x + y = -21 \text{ By subtraction}$$

$$x + y = -5$$

$$\hline 2x = -16$$

$$x = -8$$

**Step 2: Solve for  $y$**

To find the value of  $y$  substitute  $-8$  in the equation  $x + y = -5$ , then we have

$$-8 + y = -5, y = 3$$

#### **Example 2.**

Solve by elimination

$$-2x + y = 4$$

$$-2x + 2y = 0$$

**Step 1:** Eliminate  $x$  using subtraction.

$$-2x + y = 4$$

$$\hline -2x + 2y = 0$$

$$-y = 4 \text{ or } y = -4$$

**Step 2: Solve for  $x$**

To find the value of  $x$  substitute  $-4$  in the equation  $-2x + y = 4$ , then we have

$$-2x + (-4) = 4, x = -4$$

#### **Example 3.**

$$4x + 3y = -13 \text{ (equation 1)}$$

$$-5x + 2y = -24 \text{ (equation 2)}$$

**Step 1:** To solve this system of equations we must first determine which variable is easier to eliminate. It appears that it is easier to eliminate y. To do this multiply equation 1 by 2 and equation 2 by 3.

$$\begin{array}{r} 8x + 6y = -26 \\ -15x + 6y = -72 \\ \hline \end{array}$$

**Step 2:** Eliminate y using subtraction.

$$\begin{array}{r} 8x + 6y = -26 \\ -15x + 6y = -72 \\ \hline 23x = 46 \\ x = 2 \end{array}$$

**Step 3: Solve for y**

To find the value of y substitute 2 in the equation  $4x + 3y = -13$ , then we have  $4(2) + 3y = -13$ ,  $y = -7$

### Lesson 1 Exercise

Solve each system of equations using substitution method.

1)  $5x + 2y = 16$   
 $x + 8y = 26$

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2)  $c + 6d = 7$   
 $-c - 2d = -2$

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3)  $8p + 7q = 43$   
 $2p - 7 = -q$

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4)  $-5a + b = 8$   
 $7a + 9b = -32$

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5)  $-5 = 2m + 6n$   
 $4m + 5n - 18 = 0$

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6)  $v = 2 - 6u$   
 $9u + 2v = 3$

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7)  $r + 2s = 4$   
 $3s + r = 1$

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8)  $6y + 5z = 0$   
 $3z = 7y + 53$

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Solve each system of equations using elimination method.

1)  $2x + 5y = 20$   
 $6x - 5y = 12$

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2)  $3p + 4q = -3$   
 $-p + 4q = -15$

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3)  $-7a + 3b = 15$   
 $7a - 6b = -3$

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4)  $u + v = 11$   
 $-u + v = 9$

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5)  $5r + 8s = 3$   
 $4r + 8s = -4$

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6)  $-9c - 2d = 8$   
 $-9c - d = 6$

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7)  $m - 4n = 13$   
 $m - 6n = 12$

8)  $3s + 7t = 18$   
 $3s - 4t = -48$