LESSON 2

FACTORING COMPLICATED POLYNOMIALS

Factoring complicated polynomials is very different from factoring other polynomials. There are several methods to perform this task as will be shown in the following examples.

Example 1.

$x^3 - x^2 + 4x - 4$
$(x^3 - x^2) + (4x - 4)$
$x^2(x - 1) + 4(x - 1)$
$(x - 1)(x^2 + 4)$

Example 2.

$100x^2 - 16y^2$ This is the Difference of Squares. The general form is $a^2 - b^2 = (a - b)(a + b)$.
$(10x - 4y)(10x + 4y)$

Example 3.

$9x^4 - 243x$ Factor the expression by the GCF, 9x. What is left is the Difference of Two Cubes.
$9x(x^3 - 27)$ The general form is $(a - b)(a^2 + ab + b^2)$
$9x(x - 3)(x^2 + 3x + 9)$

Example 4.

$4y^5 + 32y^2$ Factor the expression by the GCF, $4x^2$. What is left is the Sum of Two Cubes
$4y^2(y^3 + 8)$ The general form is $(a + b)(a^2 - ab + b^2)$
$4y^2(y + 2)(y^2 - 2y + 4)$

Example 5.

$(p - q)^2 - 6(p - q) - 16$ This should be factored as a trinomial.
$\{(p - q) - 8\}\{(p - q) + 2\}$

Example 6

$(a + b)^2 + 9(a + b) + 18$ This should be factored as a trinomial
$\{(a + b) + 3\} + \{(a + b) + 6\}$

Lesson 2 Exercise

1. $27x^5 - 729x^2$
2. $3x^4 - 243$
3. $2x^3 - 16$
4. $5x^3 - 10x^2 + 3x - 6$
5. $x^5 - 4x^3 - x^2 + 4$
6. $x^6 - 64$
7. $6x^2 + 2xy - 3xz - yz$
8. $(x + 3)^2 - 2(x + 3) - 35$
9. $6(2p + q)^2 - 5(2p + q) - 25$
10. $44x^3 - 99y^3$
11. $512 - 216y^3$
12. $686y^3 + 250z^3$
13. $192a^5y - 648a^2y^4$
14. $16a^4 - 81b^8$

SOLUTIONS

Lesson 1 Exercise

1. $14x^3 - 10x^2 + 21x - 15$
   $(14x^3 - 10x^2) + (21x - 15)$
   $2x^2(7x - 5) + 3(7x - 5)$
   $(7x - 5)(2x^2 + 3)$

2. $2x^3 - 5x^2 + 16x - 40$
   $(2x^3 - 5x^2) + (16x - 40)$
   $x^2(2x - 5) + 8(2x - 5)$
   $(2x - 5)(x^2 + 8)$

3. $20b^3 + 25b^2 - 28b - 35$
   $(20b^3 + 25b^2) - (28b + 35)$
   $5b^2(4b + 5) - 7(4b + 5)$
   $(4b + 5)(5b^2 + 7)$

4. $35a^3 - 56a^2 - 10a + 16$
   $(35a^3 - 56a^2) - (10a + 16)$
   $7a^2(5a - 8) - 2(5a - 8)$
   $(5a - 8)(7a - 2)$

5. $30k^3 + 35k^2 + 24k + 28$
   $(30k^3 + 35) + (24k + 28)$
   $5k^2(6k + 7) + 4(6k + 7)$
   $(6k + 7)(5k^2 + 4)$

6. $14v^3 + 49v^2 - 4v - 14$
   $(14v^3 + 49v^2) - (4v - 14)$
   $7v^2(2v + 7) - 2(2v + 7)$
   $(2v + 7)(7v^2 - 2)$

7. $8p^3 + 56p^2 - 7p - 49$
   $(8p^3 + 56p^2) - (7p + 49)$
   $8p^2(p + 7) - 7(p + 7)$
   $(p + 7)(8p^2 - 7)$

8. $n^3 - 2n^2 - 4n + 8$
   $(n^3 - 2n^2) - (4n - 8)$
   $n^2(n - 2) - 4(n - 2)$
   $(n - 2)(n^2 - 4)$
   $(n - 2)(n - 2)(n + 2)$

Lesson 2 Exercise

1. $27x^5 - 729x^2$
   $x^2(27x^3 - 729)$
   $x^2(3x - 9)(9x^2 + 27x + 81)$

2. $3x^4 - 243$
   $3(x^4 - 81)$
   $3(x^2 - 9)(x^2 + 9)$
   $3(x - 3)(x + 3)(x^2 + 9)$

3. $2x^3 - 16$
   $2(x^3 - 8)$
   $2(x - 2)(x^2 + 2x + 4)$

4. $2x^4 - 58x^2 + 200$
   $2(x^4 - 29x^2 + 100)$
   $2(x^2 - 4)(x^2 - 25)$
   $2(x - 2)(x + 2)(x - 5)(x + 5)$

5. $x^5 - 4x^3 - x^2 + 4$
   $(x^5 - 4x^3) - (x^2 - 4)$
   $x^3(x^2 - 4) - 1(x^2 - 4)$
   $(x^3 - 1)(x^2 - 4)$
   $(x - 1)(x^2 + x + 1)(x - 2)(x + 2)$

6. $x^6 - 64$
   $(x^3 - 8)(x^3 + 8)$
   $(x - 2)(x^2 + 2x + 4)(x +2)(x^2 - 2x + 4)$

7. $6x^2 + 2xy - 3xz - yz$
   $(6x^2 + 2xy) - (3xz - yz)$
   $2x(3x + y) - z(3x + y)$
   $(2x - z)(3x + y)$

8. $(x + 3)^2 - 2(x + 3) - 35$
   $\{(x + 3) - 7\}\{(x + 3) + 5\}$

9. $6(2p + q)^2 - 5(2p + q) - 25$
   $\{3(2p + q) + 5\}\{2(2p + q) - 5\}$

10. $44x^3y - 99xy^3$
    $11xy(4x^2 - 9y^2)$
    $11xy(2x - 3y)(2x + 3y)$

11. $512 - 216y^3$
    $8(64 - 27y^3)$
    $8(4 - 3y)(16 + 12y + 9y^2)$

12. $686y^3 + 250z^3$

    $2(343y^3 + 125z^3)$

    $2(7y + 5z)(49y^2 - 35yz + 25z^2)$

13. $192a^5y - 648a^2y^4$

    $3a^2y(64a^3 - 216y^3)$

    $3a^2y(4a - 6y)(16a^2 + 24ay + 36y^2)$

14. $16a^4 - 81b^8$

    $(4a^2 - 9b^4)(4a^2 + 9b^4)$

    $(2a - 3b^2)(2a + 3b^2)(4a^2 + 9b^4)$