LESSON 1

FACTORING BY GROUPING

Factoring out the Greatest Common Factor (GCF)

Factoring is a technique that is useful when trying to solve polynomial equations algebraically. We begin by looking for the Greatest Common Factor (GCF) of a polynomial expression. The GCF is the largest monomial that divides (is a factor of) each term of the polynomial.

Example 1.

$$3y^4 + 9y^2 - 6y^3 - 18y$$

$$= 3y\left[y^3 + 3y - 2y^2 - 6\right] \qquad \textit{Factor out the GCF.}$$

$$= 3y\left[y(y^2 + 3) - 2(y^2 + 3)\right] \qquad \textit{Factor by grouping.}$$

$$= 3y\left[(y^2 + 3)(y - 2)\right]$$

$$= 3y(y^2 + 3)(y - 2)$$

Factoring when there is no GCF for all the terms

First group the first two terms together, then group the last two terms together, next factor out a GCF from each separate binomial, then factor out the common binomial.

Example 2.

1. Divide the polynomial into two groups: 1st half and 2nd half.

   $2x^3 - 10x^2 \mid + 3x - 15$

2. Factor the GCF out of the 1st half and factor the GCF out of the 2nd half.

   $2x^2 (x - 5) + 3(x - 5)$

3. You should have a common binomial/trinomial factor.

   $2x^2 (x - 5) + 3 (x - 5)$

4. Factor out the common binomial/trinomial factor.

   $(x - 5) (2x^2 + 3)$

Example 3.

In the example below there is no GCF for the polynomial so we divide it into two parts as shown.

$$6n^3 + 3n^2 + 8n + 4$$
$$(6n^3 + 3n^2)(+8n + 4)$$
$$3n^2(2n + 1) + 4(2n + 1)$$
$$(3n^2 + 4)(2n + 1)$$

After dividing the polynomial, factor the first part by $3n^2$ and the second part by 4. So, we now have the third line as shown above. Factoring the common binomial factor, we obtain the fourth line above.

Lesson 1 Exercise

Factor each completely.

1) $14x^3 - 10x^2 + 21x - 15$

2) $2x^3 - 5x^2 + 16x - 40$

3) $20b^3 + 25b^2 - 28b - 35$

4) $35a^3 - 56a^2 - 10a + 16$

5) $30k^3 + 35k^2 + 24k + 28$

6) $14v^3 + 49v^2 - 4v - 14$

7) $8p^3 + 56p^2 - 7p - 49$

8) $n^3 - 2n^2 - 4n + 8$