

ALGEBRA 2

Solving Harder Algebraic Equations

Problems

2) $x^4 - 17x^2 + 16 = 0$

3) $x^4 - 25x^2 + 144 = 0$

4) $x^4 - 13x^2 + 36 = 0$

5) $4x^4 - 101x^2 + 25 = 0$

6) $25x^4 - 32x^2 + 7 = 0$

7) $x^8 + 8x^4 - 9 = 0$

8) $x^8 - 18x^4 + 81 = 0$

9) $9x^4 - 61x^2 - 100 = 0$

10) $3x^4 + 112x^2 - 144 = 0$

11) $6x^8 + 58x^4 - 64 = 0$

12) $4x^4 - 37x^2 + 9 = 0$

13) $x^2 + 9x - 36 = 0$

14) $4x^2 + 6x - 70 = 0$

15) $x^4 + 6x^2 - 40 = 0$

16) $\frac{5}{x} + \frac{x+5}{5} = \frac{39}{5x}$

SOLUTIONS

1) $x^4 - 17x^2 + 16 = 0$

$$(x^2 - 1)(x^2 - 16) = 0$$

$$(x - 1)(x + 1)(x - 4)(x + 4) = 0$$

$$x - 1 = 0 \quad x + 1 = 0 \quad x - 4 = 0 \quad x + 4 = 0$$

$$x = 1 \quad , \quad x = -1 \quad , \quad x = 4 \quad , \quad x = -4$$

2) $x^4 - 25x^2 + 144 = 0$

$$(x^2 - 9)(x^2 - 16) = 0$$

$$(x - 3)(x + 3)(x - 4)(x + 4) = 0$$

$$x - 3 = 0 \quad x + 3 = 0 \quad x - 4 = 0 \quad x + 4 = 0$$

$$x = 3 \quad , \quad x = -3 \quad , \quad x = 4 \quad , \quad x = -4$$

3) $x^4 - 13x^2 + 36 = 0$

$$(x^2 - 4)(x^2 - 9) = 0$$

$$(x - 2)(x + 2)(x - 3)(x + 3) = 0$$

$$x - 2 = 0 \quad x + 2 = 0 \quad x - 3 = 0 \quad x + 3 = 0$$

$$x = 2 \quad x = -2 \quad x = 3 \quad x = -3$$

4) $4x^4 - 101x^2 + 25 = 0$

$$(4x^2 - 1)(x^2 - 25) = 0$$

$$(2x - 1)(2x + 1)(x - 5)(x + 5) = 0$$

$$2x - 1 = 0 \quad 2x + 1 = 0$$

$$2x = 1 \quad 2x = -1 \quad x - 5 = 0 \quad x + 5 = 0$$

$$x = \frac{1}{2} \quad , \quad x = -\frac{1}{2} \quad , \quad x = 5 \quad , \quad x = -5$$

5) $25x^4 - 32x^2 + 7 = 0$

$$(25t^2 - 7)(t^2 - 1) = 0$$

$$(25t^2 - 7)(t - 1)(t + 1) = 0$$

$$25t^2 - 7 = 0$$

$$25t^2 = 7$$

$$t^2 = \frac{7}{25}$$

$$t = \frac{\sqrt{7}}{5} \quad t - 1 = 0 \quad t + 1 = 0$$
$$t = 1 \quad t = -1$$

$$6) \quad x^8 + 8x^4 - 9 = 0$$

$$(x^4 - 1)(x^4 + 9) = 0$$

$$(x^2 - 1)(x^2 + 1)(x^4 + 9) = 0$$

$$x^2 - 1 = 0 \quad x^2 + 1 = 0 \quad x^4 + 9 = 0$$

$$x^2 = 1 \quad x^2 = -1 \quad x^4 = -9$$

$$x = \pm 1, \quad x = \pm i, \quad x = \pm i\sqrt[4]{9}$$

$$7) \quad x^8 - 18x^4 + 81 = 0$$

$$(x^4 - 9)(x^4 - 9) = 0$$

$$(x^2 - 3)(x^2 + 3)(x^2 - 3)(x^2 + 3) = 0$$

$$x^2 - 3 = 0 \quad x^2 + 3 = 0$$

$$x^2 = 3 \quad x^2 = -3$$

$$x = \pm\sqrt{3}, \quad x = \pm i\sqrt{3}$$

$$8) \quad 9x^4 - 61x^2 - 100 = 0$$

$$(9x^2 - 25)(x^2 - 4) = 0$$

$$(3x - 5)(3x + 5)(x - 2)(x + 2) = 0$$

$$3x - 5 = 0 \quad 3x + 5 = 0$$

$$3x = 5 \quad 3x = -5$$

$$x = \frac{5}{3}, \quad x = -\frac{5}{3}, \quad x - 2 = 0 \quad x + 2 = 0$$
$$x = 2, \quad x = -2$$

$$9) \quad 3x^4 + 112x^2 - 144 = 0$$

$$(3x^2 + 4)(x^2 - 36) = 0$$

$$(3x^2 + 4)(x - 6)(x + 6) = 0$$

$$3x^2 + 4 = 0$$

$$3x^2 = -4$$

$$x^2 = -\frac{4}{3}$$

$$x = \pm i\sqrt{\frac{4}{3}}, \quad x - 6 = 0 \quad x + 6 = 0$$
$$x = 6, \quad x = -6$$

$$10) \quad 6x^8 + 58x^4 - 64 = 0$$

$$(6x^4 + 64)(x^4 - 1) = 0$$

$$2(3x^4 + 32)(x^2 - 1)(x^2 + 1) = 0$$

$$3x^4 + 32 = 0$$

$$3x^4 = -32$$

$$x^4 = -\frac{32}{3} \quad x^2 - 1 = 0 \quad x^2 + 1 = 0$$
$$x^2 = 1 \quad x^2 = -1$$

$$x = \pm i\sqrt[4]{\frac{32}{3}}, \quad x = \pm 1, \quad x = \pm i$$

$$11) \quad 4x^4 - 37x^2 + 9 = 0$$

$$(4x^2 - 1)(x^2 - 9) = 0$$

$$(2x - 1)(2x + 1)(x - 3)(x + 3) = 0$$

$$2x - 1 = 0 \quad 2x + 1 = 0$$

$$2x = 1 \quad 2x = -1 \quad x - 3 = 0$$

$$x = \frac{1}{2}, \quad x = -\frac{1}{2}, \quad x = 3$$

$$12) x^2 + 9x - 36 = 0$$

$$(x+12)(x-3) = 0$$

$$x+12 = 0 \quad x-3 = 0$$

$$x = -12 \quad x = 3$$

$$13) 4x^2 + 6x - 70 = 0$$

$$(2x+10)(2x-7) = 0$$

$$2x - 7 = 0$$

$$2x + 10 = 0 \quad 2x = 7$$

$$2x = -10 \quad x = \frac{7}{2}$$

$$x = -5 \quad ,$$

$$14) x^4 + 6x^2 - 40 = 0$$

$$(x^2 + 10)(x^2 - 4) = 0$$

$$x^2 + 10 = 0 \quad x^2 - 4 = 0$$

$$x^2 = -10 \quad x^2 = 4$$

$$x = \pm i\sqrt{10} \quad , \quad x = \pm 2$$

$$15) \frac{5}{x} + \frac{x+5}{5} = \frac{39}{5x}$$

$$25 + x^2 + 5x = 39$$

$$x^2 + 5x - 14 = 0$$

$$(x+7)(x-2) = 0$$

$$x+7 = 0 \quad x-2 = 0$$

$$x = -7 \quad , \quad x = 2$$