ALGEBRA 2

STANDARD DEVIATION

How to find the standard deviation without a calculator.

The formula for the population standard deviation is, $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x-\mu)^2}{n}}$.

The formula for the sample standard deviation is, $s = \sqrt{\dfrac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$.

Example 1. Standard Deviation of a Population.

x	μ	x - μ	(x - μ)²
5	7	-2	4
5	7	-2	4
7	7	0	0
9	7	2	4
6	7	-1	1
10	7	3	9
4	7	-3	9
8	7	1	1
9	7	2	4
7	7	0	0
Σx = 70			Σ(x - μ)² = 36

Assume that the ages of 10 children in a party are 5, 5, 7, 9, 6, 10, 4, 8, 9, and 7. To find the standard deviation of this population we must first find the mean. Next find the difference of each data point from the mean, square each result, and divide the sum by the number of the data points. The method is clearly shown in the example below.

Working.

$\mu = \frac{\sum x}{n} = \frac{70}{10} = 7$. $\Sigma(x-\mu)^2 = 36$. Therefore, $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x-\mu)^2}{n}} = \sqrt{\frac{36}{10}} = 1.9$

The standard deviation of the ages of the children is 1.9 years.

Example 2. Standard Deviation of a Sample.

The sample results of 8 students in a college class of 60 shows the following grades: 95, 60, 35, 25, 80, 70, 65, 75, 85, and 90. To find the standard deviation of this sample we must first find the mean. Next find the difference of each data point from the mean, square each result, and divide the sum by one less (n - 1) than the number of the data points. The method is clearly shown in the example below.

X	$\bar{X}$	X - $\bar{X}$	(X - $\bar{X}$)^2
95	65	30	900
60	65	-5	25
35	65	-30	900
20	65	-45	2025
80	65	15	225
70	65	5	25
75	65	10	100
85	65	20	400
Σx = 520			Σ(X - $\bar{X}$)^2 = 4600

Working.

$\bar{X} = \frac{\sum x}{n} = \frac{520}{8} = 65$. $\sum\left(X - \bar{X}\right)^2 = 4600$.

Therefore, $s = \sqrt{\frac{\sum\left(X - \bar{X}\right)^2}{n-1}} \sqrt{\frac{4600}{7}} = \sqrt{657.143} = 25.6$

The standard deviation of the students' grades is 25.6

Example 3. Standard Deviation of Grouped Data.

The grades for the 40 students in the Economics II class at St. Joeval College is grouped. The result is shown below.

Grades	40-49	50-59	60-69	70-79	80-89	90-100
frequency	4	3	6	9	10	8

The formula for Grouped Data is: $s = \sqrt{\frac{\sum f\left(x - \bar{X}\right)^2}{\sum f - 1}}$.

To find the standard deviation of grouped data we must first find the midpoint of each class. Next find the mean by summing the product of each midpoint and its' frequency, then divide the result by the sum of the frequencies, $\left(\frac{\sum fx}{\sum f}\right)$. Find the difference of each midpoint and the mean and square each result; finally, divide the sum by one less (Σf-1) than the number of the data points. The method is clearly shown in the example below.

Grade	f	x	fx	$\bar{X}$	x- $\bar{X}$	(x- $\bar{X}$)^2	f(x- $\bar{X}$)^2
40-49	4	44.5	178.0	75	-30.5	930.25	3721.00
50-59	3	54.5	163.5	75	-20.5	420.25	1260.75

60-69	6	64.5	387.0	75	-10.5	110.25	661.50
70-79	9	74.5	670.5	75	-0.5	0.25	2.25
80-89	10	84.5	845.0	75	9.5	90.25	902.50
90-100	8	95	760.0	75	20	400	3200.00
	Σf=40		$\sum fx$ =3004				$\sum f\left(x-\bar{X}\right)^2$ =9748

Working.

$\bar{X} = \frac{\sum fx}{\sum f} = \frac{3004}{40} = 75$, $\sum f\left(x-\bar{X}\right)^2$ =9748,

Therefore, $s = \sqrt{\frac{\sum f\left(x-\bar{X}\right)^2}{\sum f - 1}} = \sqrt{\frac{9748}{39}} = \sqrt{249.949} = 15.81$

The standard deviation for students' grades could be rounded to 15.8.