USING DOUBLE ANGLE FORMULAS TO ESTABLISH TRIGONOMETRIC IDENTITIES.

1) $\cos^4\theta - \sin^4 = \cos(2\theta)$

2) $\cot(2\theta) = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

3) $\sec(2\theta) = \dfrac{\sec^2\theta}{2 - \sec^2\theta}$

4) $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$

5) $\dfrac{\cos(2\theta)}{1 + \sin(2\theta)} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

6) $\dfrac{\sin(3\theta)}{\sin\theta} - \dfrac{\cos(3\theta)}{\cos\theta} = 2$

7) $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

8) $\cot(2\theta) = \dfrac{1}{2}(\cot\theta - \tan\theta)$

9) $\dfrac{\tan(2\theta) + \cot(2\theta)}{\csc(2\theta)} = \sec(2\theta)$

10) $\tan^2(2x) + \sin^2(2x) + \cos^2(2x) = \sec^2(2x)$

11) $\cos(3\theta) = 4\cos^2\theta - 3\cos\theta$

12) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

13) $\sin(4t) = 4\sin t\cos^3 t - 4\sin^3 t\cos t$

14) $\cot x = \dfrac{\sin 2x}{1 - \cos 2x}$

15) $\sin(2t) - \tan t = \tan t\cos(2t)$

16) $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$

SOLUTIONS TO PROBLEMS

1) $\cos^4\theta - \sin^4 = \cos(2\theta)$

LHS $= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= [\cos(2\theta)][1] = \cos(2\theta)$

2) $\cot(2\theta) = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

$$= \frac{1-\tan^2\theta}{2\tan\theta} = \frac{1-\tan^2\theta}{2\tan\theta} \times \frac{\frac{1}{\tan^2\theta}}{\frac{1}{\tan^2\theta}}$$

$$= \frac{\frac{1}{\tan^2\theta} - \frac{\tan^2\theta}{\tan^2\theta}}{\frac{2\tan\theta}{\tan^2\theta}} = \frac{\cot^2\theta - 1}{2\cot\theta}$$

3) $\sec(2\theta) = \dfrac{\sec^2\theta}{2-\sec^2\theta}$

LHS $= \dfrac{1}{\cos(2\theta)} = \dfrac{1}{2\cos^2\theta - 1}$

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}} = \frac{\sec^2\theta}{2-\sec^2\theta}$$

4) $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$

$= [\cos(2\theta)][\cos(2\theta)] - [\sin(2\theta)][\sin(2\theta)]$

$= \cos(4\theta)$

5) $\dfrac{\cos(2\theta)}{1+\sin(2\theta)} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

LHS $\quad = \dfrac{\cos^2\theta - \sin^2\theta}{1-\sin(2\theta)} = \dfrac{(\cos\theta+\sin\theta)(\cos\theta-\cos\theta)}{\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta}$

$$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta+\sin\theta)^2} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

RHS $\quad \left(\dfrac{\cot\theta-1}{\cot\theta+1} \Rightarrow \dfrac{\dfrac{\cos\theta}{\sin\theta}-1}{\dfrac{\cos\theta}{\sin\theta}+1} \Rightarrow \dfrac{\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\sin\theta}} \Rightarrow \dfrac{\dfrac{\cos\theta-\sin\theta}{\sin\theta}}{\dfrac{\cos\theta+\sin\theta}{\sin\theta}} \right)$

$$= \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

6) $\dfrac{\sin(3\theta)}{\sin\theta} - \dfrac{\cos(3\theta)}{\cos\theta} = 2$

$$= \frac{\sin(2\theta+\theta)}{\sin\theta} - \frac{\cos(2\theta+\theta)}{\cos\theta}$$

$$= \left(\frac{\sin(2\theta)\cos\theta+\cos(2\theta)\sin\theta}{\sin\theta}\right) - \left(\frac{\cos(2\theta)\cos\theta-\sin(2\theta)\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{2\sin\theta\cos\theta\cos\theta+\sin\theta(\cos^2\theta-\sin^2\theta)}{\sin\theta}\right) - \left(\frac{\cos\theta(\cos^2\theta-\sin^2\theta)-\sin\theta\cos\theta\sin\theta}{\cos\theta}\right)$$

$$= 2\cos^2\theta + (\cos^2\theta-\sin^2\theta) - (\cos^2\theta-\sin^2\theta) + 2\sin^2\theta$$

$$= 2\cos^2\theta + 2\sin^2\theta$$

$$= 2(\cos^2\theta+\sin^2\theta)$$

$$= 2(1) = 2$$

7) $\dfrac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta}=\cos(2\theta)$

LHS $$=\frac{\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}}\Rightarrow\frac{\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}$$

$$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}\Rightarrow\frac{\cos(2\theta)}{1}=\cos(2\theta)$$

8) $\cot(2\theta)=\dfrac{1}{2}(\cot\theta-\tan\theta)$

LHS $$=\frac{1}{\tan(2\theta)}\Rightarrow\frac{1}{\dfrac{2\tan(2\theta)}{1-\tan^2(2\theta)}}\Rightarrow\frac{1-\tan^2(2\theta)}{2\tan(2\theta)}\times\frac{\dfrac{1}{\tan(2\theta)}}{\dfrac{1}{\tan(2\theta)}}$$

$$=\frac{\dfrac{1}{\tan(2\theta)}-\tan(2\theta)}{2}\Rightarrow\frac{1}{2}\left(\cot\theta-\tan\theta\right)$$

9) $\dfrac{\tan(2\theta)+\cot(2\theta)}{\csc(2\theta)}=\sec(2\theta)$

LHS $$=\frac{\dfrac{\sin(2\theta)}{\cos(2\theta)}+\dfrac{\cos(2\theta)}{\sin(2\theta)}}{\dfrac{1}{\sin(2\theta)}}\Rightarrow\frac{\dfrac{\sin^2(2\theta)+\cos^2(2\theta)}{\cos(2\theta)\sin(2\theta)}}{\dfrac{1}{\sin(2\theta)}}\Rightarrow\frac{\sin^2(2\theta)+\cos^2(2\theta)}{\cos(2\theta)}$$

$$=\frac{1}{\cos(2\theta)}\Rightarrow\sec(2\theta)$$

10) $\tan^2(2x)+\sin^2(2x)+\cos^2(2x)=\sec^2(2x)$

LHS $$=\tan^2(2x)+[\sin^2(2x)+\cos^2(2x)]\Rightarrow\tan^2(2x)+1\Rightarrow\frac{\sin^2(2x)}{\cos^2(2x)}+1$$

$$= \frac{\sin^2(2x) + \cos^2(2x)}{\cos^2(2x)} \Rightarrow \frac{1}{\cos^2(2x)} \Rightarrow \sec^2(2x)$$

11) $\cos(3\theta) = 4\cos^2\theta - 3\cos\theta$

$$= \cos(2\theta + \theta) \Rightarrow \cos(2\theta)\cos\theta - \sin(2\theta + \theta)$$

LHS $= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta \Rightarrow 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$

$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta \Rightarrow 4\cos^3\theta - 3\cos\theta$$

12) $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

LHS $= \sin(2\theta + \theta) \Rightarrow \sin(2\theta)\cos\theta + \cos(2\theta)\sin\theta$

$$= (2\sin\theta\cos\theta)\cos\theta + (2\cos^2\theta - 1)\sin\theta$$

$$= 2\sin\theta\cos^2\theta + 2\sin\theta\cos^2\theta - \sin\theta$$

$$= 4\sin\theta\cos^2 - \sin\theta \Rightarrow 4\sin\theta(1 - \sin^2\theta) - \sin\theta$$

$$= 4\sin\theta - 4\sin^3\theta - \sin\theta \Rightarrow 3\sin\theta - 4\sin^3\theta$$

13) $\sin(4t) = 4\sin t\cos^3 t - 4\sin^3 t\cos t$

LHS $= \sin(2t + 2t) \Rightarrow \sin 2t\cos 2t + \cos 2t\sin 2t$

$$= \cos 2t(\sin 2t + \sin 2t) \Rightarrow (\cos 2t)(2\sin 2t)$$

$$= (\cos^2 t - \sin^2 t)(2\sin t\cos t)(2) \Rightarrow 4\sin t\cos^3 t - 4\sin^3 t\cos t$$

14) $\cot x = \frac{\sin 2x}{1 - \cos 2x}$

RHS $\frac{\sin 2x}{1 - \cos 2x} = \frac{2\sin x\cos x}{1 - (\cos^2 x - \sin^2 x)} \Rightarrow \frac{2\sin x\cos x}{1 - \cos^2 x + \sin^2 x}$

$$=\frac{2\sin x\cos x}{\sin^2 x+\sin^2 x}\Rightarrow\frac{2\sin x\cos x}{2\sin^2 x}\Rightarrow\frac{\cos}{\sin x}\Rightarrow\cot x$$

15) $\sin(2t)-\tan t=\tan t\cos(2t)$

RHS $\tan t\cos 2t=\frac{\sin t}{\cos t}(2\cos^2 t-1)\Rightarrow\frac{2\sin t\cos^2 t}{\cos t}-\frac{\sin t}{\cos t}$

$$=2\sin t\cos t-\tan t\Rightarrow\sin 2t-\tan t$$

16) $\cos^2(2\theta)-\sin^2(2\theta)=\cos(4\theta)$

$$\cos(4\theta)=\cos(2\theta+2\theta)\Rightarrow\left(\cos(2\theta)\cos(2\theta)\right)-\left(\sin(2\theta)\sin(2\theta)\right)$$

RHS $=\cos^2(2\theta)-\sin^2(2\theta)$