ALGEBRA 2

CONIC SECTIONS - PARABOLAS

Identify the vertex, focus, axis of symmetry, and directrix of each of the following.

1. $y = -(x+4)^2 - 2$
2. $x = -\frac{1}{4}(y+3)^2$
3. $-\frac{1}{3}(x-3) = (y+5)^2$
4. $-(y+2) = (x-1)^2$
5. $3y + 4x = -2x^2 - 14$
6. $x = -2(y+2)^2$
7. $-2x^2 - 4x + 2y + 70 = 0$
8. $4y^2 + x + 24y + 51 = 0$
9. $-4(y+2) = (x+8)^2$
10. $(y+3)^2 = 8(x-1)$

Write the transformational form of the equation using the given 3 points. The graph opens up or down.

11. Points are: (-4, -3), (-9, 27), and (-3, 3).

12.Points are: (-3, -4), (2, 6), and (-4, 6).

13. Points are: (-6, -3), (-2, -11), and (-10, -3).

14. Points are: (-3, 12), (3, -12), (-15, -12)

Write the equation for each parabola with the given focus, point on the graph, and orientation.

15. F: (-5, 1.5), point: (-9, 9), opens up.

16. F: (-3, -2), point: (9, -11), opens down.

17. F: (4, 3), point: (-4, 9), opens up.

18. F: (-3, -3), point: (-11, -3), opens down.

19. F: (6, 10), point: (10, 7), opens down.

20. F: (-8, -3), point: (-2, -11), opens down.

Solutions

Identify the vertex, focus, axis of symmetry, and directrix of each of the following.

Note: (1) Parabola opening vertically.

Vertex: (h, k)

Focus: (h, k + p)

Directrix: y = k – p

Axis of symmetry: x = h

(2) Parabola opening horizontally.

Vertex: (h, k)

Focus: (h + p, k)

Directrix: x = h – p

Axis of symmetry: y = k

(3) The parabola, $4p(y - k) = (x - k)^2$ opens up or down.

(4) The parabola, $4p(x - h) = (y - k)^2$ opens left or right.

1.

$$y = -(x+4)^2 - 2$$
$$(y+2) = -(x+4)^2$$
$$-1(y+2) = (x+4)^2$$

4p = -1, p = -1/4

Vertex: (-4,-2), Focus: (-4, -9/4), Axis of Symmetry: x = -4, Directrix: y = -7/4.

2.

$$x = -\frac{1}{4}(y+3)^2$$
$$-4x = (y+3)^2$$

4p = -4 and p = -1

Vertex: (0, -3), Focus: (-1, -3), Axis of symmetry: y = -3, Directrix: x =1.

3 $-\frac{1}{3}(x-3)=(y+5)^2$.

4p = -1/3, p = -1/12

Vertex: (3, -5), Focus: (35/12, -5), Axis of symmetry: y = -5, Directrix: *x* = 37/12.

4. $-(y+2)=(x-1)^2$

4p = -1, p = -1/4

Vertex: (1, -2), Focus: (1, -9/4), Axis of symmetry: *x* = 1, Directrix: -7/4.

5.

$$3y+4x=-2x^2-14$$
$$2x^2+4x=-3y+14$$
$$2\left(x^2+2x+1\right)=-3y-14+2$$
$$2(x+1)^2=-3(y+4)$$
$$(x+1)^2=-3/2(y+4)$$

4p = -3/2, p = -3/8

Vertex: (-1, -4), Focus: (-1, -35/8), Axis of symmetry: *x* = -1, Directrix: y = -29/8

6.

$$x = -2(y+2)^2$$
$$-1/2x = (y+2)^2$$

4p = -1/2, p = -1/8

Vertex: (0, -2), Focus: (-1/8, -2), Axis of Symmetry: y = -2, Directrix: x = 1/8.

7.

$$-2x^2 - 4x + 2y + 70 = 0$$
$$-2(x^2 + 2x + 1) = -2y - 70 - 2$$
$$-2(x+1)^2 = -2y - 72$$
$$-2(x+1)^2 = -2(y+36)$$
$$(x+1)^2 = (y+36)$$

4p = 1, p = ¼

Vertex: (-1, -36), Focus: (-1, -143/4), Axis of Symmetry: x = -1, Directrix: y = -145/4.

8.

$$4y^2 + x + 24y + 51 = 0$$
$$4y^2 + 24y = -x - 51$$
$$4(y^2 + 6y + 9) = -x - 51 + 36$$
$$(y+3)^2 = -1/4(x+15)$$

4p = -1/4, p = -1/16

Vertex: (-15, -3), Focus: (-241/16, -3), Axis of symmetry: y = -3, Directrix: x = -239/16.

9. $-4(y+2)=(x+8)^2$ 4p = -4, p = -1

Vertex: (-8, -2), Focus: (-8, -3), Axis of Symmetry: x = -8, Directrix: y = -1.

10. $(y+3)^2=8(x-1)$ 4p = 8, p = 2

Vertex: (1, -3), Focus: (3, -3), Axis of symmetry: y = -3, Directrix: x = -1.

Write the transformational form of the equation using the given 3 points. The graph opens up or down.

11. Points are (-4, -3), (-9, 27), and (-3, 3).

$$4p(-3-k)=(-4-h)^2$$

Equation (1) $-12p-4kp=16+8h+h^2$

$$4p(27-k)=(-9-h)^2$$

Equation(2) $108p-4kp=81+18h+h^2$

$$4p(3-k)=(-3-h)^2$$

Equation (3) $12p-4kp=9+6h+h^2$

Equation (1) – Equation (2)

$$-12p-4kp=16+8h+h^2$$
$$108p-4kp=81+18h+h^2$$
$$-120p=-65-10h$$
$$-24p=-13-2h(\div by5)$$

Equation (4)

Equation (2) – Equation (3)

$$108p-4kp=81+18h+h^2$$
$$12p-4kp=9+6h+h^2$$
$$96p=72+12h$$
$$16p=12+2h(\div by6)$$

Equation (5)

Equation (4) + Equation (5)

$$-24p = -13 - 2h$$
$$16p = 12 + 2h$$
$$-8p = -1 \Rightarrow p = 1/8 \Rightarrow 4p = 1/2$$

Solving for h,

$$16p = 12 + 2h$$
$$16\left(\frac{1}{8}\right) = 12 + 2h$$
$$2 = 12 + 2h$$
$$-10 = 2h \Rightarrow h = -5.$$

To solve for k,

$$12p - 4kp = 9 + 6h + h^2$$
$$12\left(\frac{1}{8}\right) - 4\left(\frac{1}{8}\right)k = 9 + 6(-5) + (-5)^2$$
$$1.5 - 0.5k = 9 - 30 + 25$$
$$-0.5k = 4 - 1.5 \Rightarrow k = -5$$

4p = ½, h = -5, k = -5. Therefore, the equation is: **½(y + 5) = (x + 5)².**

12. Points are: (-3, -4), (2, 6), and (-4, 6)

Equation (1)

$$4p(-4-k) = (-1-h)^2$$
$$-16p - 4kp = 9 + 6h + h^2$$

Equation (2)

$$4p(6-k) = (2-h)^2$$
$$24p - 4kp = 4 - 4h + h^2$$

Equation(3)

$$4p(6-k) = (-4-h)^2$$
$$24p - 4kp = 16 + 8h + h^2$$

Equation (1) – Equation (2)

Equation (1) – Equation (3)

$$-16p-4kp=9+6h+h^2$$
$$24p-4kp=4-4h+h^2$$
$$-40p=5+10h$$
$$-8p=1+2h \quad \text{(Equation 4)}$$

$$-16p-4kp=9+6h+h^2$$
$$24p-4kp=16+8h+h^2$$
$$-40p=-7-2h \quad \text{(Equation 5)}$$

Equation (4) + Equation (5)

$$-8p=1+2h$$
$$-40p=-7-2h$$
$$-48p=-6 \Rightarrow p=\frac{1}{8} \Rightarrow 4p=\frac{1}{2}$$

Solving for h,

$$-8p=1+2h$$
$$-8\left(\frac{1}{8}\right)=1+2h$$
$$-1=1+2h \Rightarrow -2=2h \Rightarrow h=-1$$

Solving for k,

$$-16p-4kp=9+6h+h^2$$
$$-16\left(\frac{1}{8}\right)-4\left(\frac{1}{8}\right)k=9+6(-1)+(-1)^2$$
$$-2-0.5k=9-6+1$$
$$-0.5k=6 \Rightarrow k=-12$$

4p = ½, h =-1, k=-12. Therefore, the equation is: **½(*y* + 12) = (*x* + 1)².**

13. Points are: (-6, -3), (-2, -11), and (-10, -3).

$$4p(y-k)=(x-h).$$

$$4p(-3-k)=(-6-h)^2$$
$$-12p-4kp=36+12h+h^2\ (eq1)$$

$$4p(-3-k)=(-10-h)^2$$
$$-12p-4kp=100+20h+h^2\ (eq3)$$

$$4p(-11-k)=(-2-h)^2$$
$$-44p-4kp=4+4h+h^2\ (eq2)$$

eq(2) – eq(3),

$$-44p-4kp=4+4h+h^2\ (eq2-eq3)$$
$$-12p-4kp=100+20h+h^2$$
$$-32p=-96-16h$$
$$-2p=-6-h$$

Eq(1) – eq(2)

$$-12p-4kp=36+12h+h^2$$
$$-44p-4kp=4+4h+h^2\ (eq1-eq2)$$
$$32p=32+8h$$
$$4p=4+h$$

Solving for p,

$$4p=4+h$$
$$-2p=-6-h$$
$$2p=-2$$
$$p=-1,$$
$$4p=-4$$

Solving for h,

$$4p=4+h$$
$$4(-1)=4+h$$
$$-4-4=h$$
$$h=-8$$

Solving for k,

$$-12p-4kp=36+12k+k^2$$
$$-12(-1)-4(-1)k=36+12h+h^2$$
$$12+4k=36+12(-8)+(-8)^2$$
$$4k=36-96+64-12$$
$$4k=-8\Rightarrow k=-2$$

4p = -4, h = -8, k = -2, Equation is: **-4(*y*+2) = (*x* + 8)²**.

14. Points are: (-3, 12), (3, -12),(-15, -12).

$$4p(12-k)=(-3-h)^2$$
$$48p-4kp=9+6h+h^2\ (eq1)$$

$$4p(-12-k)=(3-h)^2$$
$$-48p-4kp=9-6h+h^2\ (eq2)$$

$$4p(-12-k)=(-15-h)^2$$
$$-48p-4kp=225+30h+h^2\ (eq3)$$

$$48p-4kp=9+6h+h^2$$
$$-48p-4kp=9-6h+h^2\ (eq1-eq2)$$
$$96p=12h$$
$$8p=h$$

$$48p-4kp=9+6h+h^2$$
$$-48p-4kp=225+30h+h^2\ (eq1-eq3)$$
$$96p=-216-24h$$
$$4p=-9-h$$

$$4p=-9-h$$
$$4p=-9-8p$$
$$12p=-9$$
$$p=-\frac{3}{4},4p=-3$$

(Finding for p.)

Solving for h.

$$96p = 12h$$
$$96\left(-\frac{3}{4}\right) = 12h$$
$$-72 = 12h$$
$$-6 = h$$

Solving for k.

$$48p - 4kp = 9 + 6h + h^2$$
$$48\left(-\frac{3}{4}\right) - 4\left(-\frac{3}{4}\right)k = 9 + 6(-6) + \left(-6^2\right)$$
$$-36 + 3k = 9 - 36 + 36$$
$$3k = 9 + 36 \Rightarrow k = 15$$

4p = -3, h = -6, k =15, therefore the equation is, **$-3(y - 15) = (x + 6)^2$.**

Write the equation for each parabola with the given focus, point on the graph, and orientation.

15. F: (-5, 1.5), point: (-9, 9), opens up.

Because the parabola opens up, the equation is, $4p(y - k) = (x - h)^2$ and the vertex is (-5, 1.5-p).

$$4p\left(9 - (1.5 - p)\right) = \left(-9 - (-5)\right)$$
$$4p(7.5 + p) = (-4)^2$$
$$30p + 4p^2 = 16$$
$$2p^2 + 15p - 8 = 0$$
$$(2p - 1)(p + 8)$$

p = ½ or -8, but the parabola opens up so p =1/2, and 4p = 2.

The vertex is, (-5, 1.5-p)

(-5, 1.5 – 0.5) = (-5, 1). The equation is, **$2(y - 1) = (x + 5)^2$**.

16. F: (-3, -2), point: (9, -11), opens down.

$4p(y - k) = (x - h)^2$

$$4p\left(-11 - (-2 - p)\right) = 144$$
$$4p(-9 + p) = 144$$
$$-36p + 4p^2 - 144 = 0$$
$$p^2 - 9p - 36 = 0$$
$$(p - 12)(p + 3)$$

p = -3 or p = 12 Since the parabola opens down, p= -3 and 4p= -12.

The vertex is: (-3, (-2- p)

(-3, -2- (-3)) = (-3, 1). Therefore, the equation is: **-12(*y* - 1) = (*x* + 3)²**.

17. F: (4, 3), point: (-4, 9), opens up.

$4p(y - k) = (x - h)^2$

$$4p\left(9-\left(3-p\right)\right)=\left(-4-4\right)^2$$
$$4p\left(6+p\right)=64$$
$$24p+4p^2-64=0$$
$$p^2+6p-16=0$$
$$\left(p-2\right)\left(p+8\right)$$

Since the parabola opens up, p = 2 and 4p = 8.

Vertex = (4, 3 – p) = (4, 1). Therefore, the equation is, **8(*y* – 1) = (*x* – 4)²**.

18. F: (-3, -3), point: (-11, -3), opens down.

$4p(y - k) = (x - h)^2$

$$4p\left(-3-\left(-3-p\right)\right)=\left(-11-\left(-3\right)\right)^2$$
$$4p^2=64$$
$$p^2-16=0$$
$$\left(p-4\right)\left(p+4\right)=0$$

Since the parabola opens down, p = -4 and 4p = -16.

Vertex = (-3, -3-p) = (-3, 1). Therefore, the equation is, **-16(*y* – 1) = (*x* + 3)².**

19. F: (6, 10), point: (10, 7), opens down.

$4p(y - k) = (x - h)^2$.

$$4p\left(7-\left(10-p\right)\right)=\left(10-6\right)^2$$
$$4p\left(-3+p\right)=16$$
$$-12p+4p^2-16=0$$
$$p^2-3p-4=0$$
$$\left(p-4\right)\left(p+1\right)=0$$

Since the parabola opens down, p = -1 and 4p = -4.

Vertex = (6, 10 – p) = (6, 11). Therefore, the equation is, **$-4(y - 11) = (x - 6)^2$**.

20. F: (-8, -3), point: (-2, -11), opens down.

$4p(y - k) = (x - h)^2$.

$$4p(-11-(-3-p))=(-2-(-8))$$
$$4p(-8+p)=36$$
$$-32p+4p^2-36=0$$
$$p^2-8p-9=0$$
$$(p-9)(p+1)$$

Since the parabola opens down, p = -1, and 4p = -4.

Vertex = (-8, -3-p) = (-8, -2). Therefore, the equation is, **$-4(y + 2) = (x + 8)^2$.**